Solution of final exam math 280 ① $\underline{5}$
first sismester 1444

Question 1

(a). A is not bounded absove.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k} = +\infty$$

so $\sup A$ D.N.E.

- $\operatorname{Inf}(A) = 1$

(b) for every $a \in A$, $a + b \geq \inf(A) + b$

Then $\inf(A+b) \geq \inf(A) + b$. ①

on the other hand $\operatorname{Inf}(A+b) - b \leq a$

for every $a \in A$, it follows

$$\operatorname{Inf}(A+b) - b \leq \inf(A)$$

$$\operatorname{Inf}(A+b) \leq \inf(A) + b \quad ②$$

① and ② show that $\operatorname{Inf}(A+b) = \operatorname{Inf}(A) + b$.

(b) $\lim_{n\to\infty} n b^n = \lim_{n\to\infty} n e^{+n\ln(b)} = 0 \; ; 0<b<1$

Question 2

(a) $\sum_{k=0}^{\infty} b_k$ by Cauchy for $\varepsilon>0$, there exists $N \in \mathbb{N}$ $(N \geq 42)$

such that, $\forall n, m \geq N$

$$\left| \sum_{k=n}^{m} b_k \right| < \varepsilon$$

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Then $\left|\sum_{k=n}^{m} u_k\right| \le \varepsilon$ for $n, m \ge N$ $(N > k_2)$

Such $a_k = b_k$ for $k \ge N$.

This gives $\sum_{k=1}^{\infty} u_k$ cv.

(b)

(1) $0 \le \frac{1}{k^2+k} \le \frac{1}{k^2}$, $k \ge N$ and $\sum_{k=}^{\infty} \frac{1}{k^2}$ cv

by comparison test $\sum_{k=1}^{\infty} \frac{1}{k^2+k}$ cv

(2). $\sum_{k=1}^{\infty} \frac{3^k+4^k}{6^k} = \sum_{k=1}^{\infty} \left(\frac{3}{6}\right)^k + \sum_{k=1}^{\infty} \left(\frac{4}{6}\right)^k < \infty$

(3) $u_k = \frac{k}{2k^2-1} \ge \frac{1}{2k}$ and $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges

then by comparison test $\sum_{k=1}^{\infty} \frac{k}{2k^2-k}$ diverges.

Question 3.

(a) $\left|\frac{1}{x} - \frac{1}{y}\right| = \frac{|x-y|}{xy}$, $x, y \ge 2$

$\le \frac{1}{4}|x-y|$

let $\varepsilon > 0$ put $\delta = 4\varepsilon$

$|x-y| < \delta \Rightarrow \left|\frac{1}{x} - \frac{1}{y}\right| \le \varepsilon$

$\frac{1}{x}$ is uniformly continuous on $\mathbb{R}$

(b)

m 1

②

(c) $x \to \cos(x)$ is continuous on $[a,b]$ and differentiable at (a,b), by MVT there exists $c \in (a,b)$ such that

$$\cos(a) - \cos b = -\sin(c)\,(b-a)$$

Then.

$$|\cos(a) - \cos(b)| \leq |b-a|, \text{ for all}$$

$a, b \in \mathbb{R}$.

(d)

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

~~if~~ $$= \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} \geq 0$$

$$= \lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} \leq 0$$

then. $f'(c) = 0$

Question 4

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} 3x - 2 = 1 = f(1)$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} x^2 = 1 = f(1)$$

then f is continuous at $x = 1$

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^-} x + 1 = 2$$

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Q 1

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{3x - 2 - 1}{x - 1} = 3.$$

the right derivative is different from the left derivative

therefore f is not differentiable at $x = 1$.

(b)

$$e = 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} \simeq$$

(c)

$$f(x) = \begin{cases} 1, & \text{if } x \in [0,1] \cap \mathbb{Q} \\ 0 & \text{if } x \notin [0,1] \cap \mathbb{Q} \end{cases}$$

f is bounded $|f(x)| \leq 1$

and f is not not integrable on $[0,1]$.

Since,

$$\overline{\int_0^1} f(x)\, dx = 1$$

$$\underline{\int_0^1} f(x)\, dx = 0$$

(d)

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