



FIRST MID TERM EXAM

Name (in Arabic): .....

Student No.: .....

Section / Instructor: .....

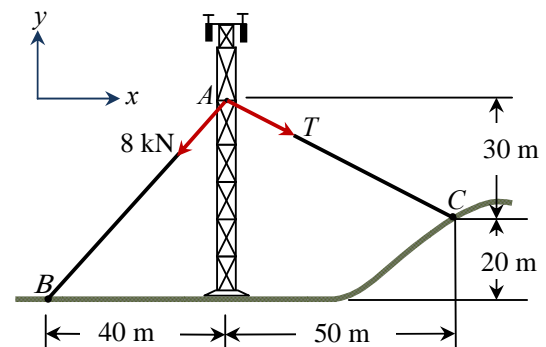
**Model  
Solution**

Q. No.	Max. Marks	Marks Obtained
1	10	
2	10	
3	10	
<b>Total</b>	<b>30</b>	

**Question # 1(a) (6 Marks)**

The cables  $AB$  and  $AC$  are attached to the top of the transmission tower as shown in the figure.  
If the tension in cable  $AB$  is 8 kN, determine

- The required tension  $T$  in the cable  $AC$  such that the resultant ( $R$ ) of the two cable tensions is along  $y$ -axis.
- The magnitude of the resultant force  $R$ .



**Solution**

The angles  $\alpha$  and  $\beta$  can be obtained from the above figure as

$$\alpha = \tan^{-1}\left(\frac{40}{50}\right) = 38.7^\circ; \text{ and } \beta = \tan^{-1}\left(\frac{50}{30}\right) = 59^\circ$$

The angle  $\gamma = 180^\circ - (\alpha + \beta) = 180^\circ - (38.7^\circ + 59^\circ) = 82.3^\circ$

Applying sine law on the second figure yields,

$$\frac{T}{\sin \alpha} = \frac{R}{\sin \gamma} = \frac{8}{\sin \beta} \Rightarrow \frac{T}{\sin 38.7^\circ} = \frac{R}{\sin 82.3^\circ} = \frac{8}{\sin 59^\circ}$$

$$\Rightarrow \frac{T}{\sin 38.7^\circ} = \frac{8}{\sin 59^\circ} \Rightarrow T = 5.83 \text{ kN} \quad \text{Ans.}$$

$$\text{Similarly, } \frac{R}{\sin 82.3^\circ} = \frac{8}{\sin 59^\circ} \Rightarrow R = 9.25 \text{ kN} \quad \text{Ans.}$$

Alternatively,

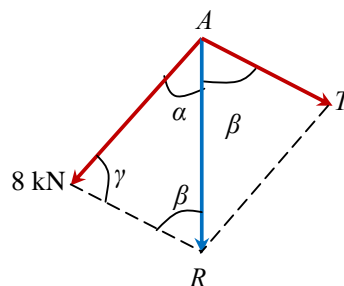
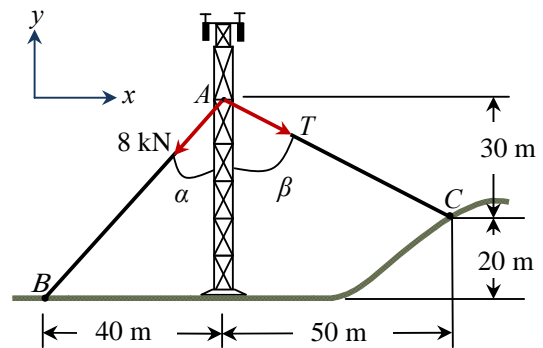
Note  $R_x = 0$ ; and  $R_y = R$

$$\rightarrow R_x = \sum F_x = T \sin \beta - 8 \sin \alpha = T \sin 59^\circ - 8 \sin 38.7^\circ = 0$$

$$\Rightarrow T = 5.83 \text{ kN} \quad \text{Ans.}$$

$$\uparrow R_y = \sum F_y = -T \cos \beta - 8 \cos \alpha = -5.83 \cos 59^\circ - 8 \cos 38.7^\circ$$

$$\Rightarrow R = R_y = -9.25 \text{ kN} \quad \text{Ans.}$$



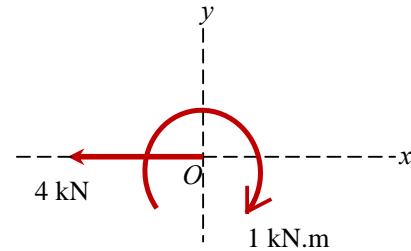
**(3 marks)**

**(3 marks)**

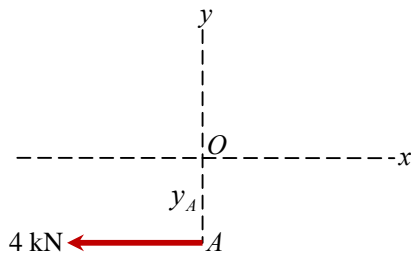
Student name		Marks obtained for Q.1	page 2/4
Student number			

**Question # 1(b) (2 Marks)**

Replace the force-couple system at point  $O$  by a single force. Specify the  $y$ -coordinate through which the line of action of this single force passes.



**Solution**



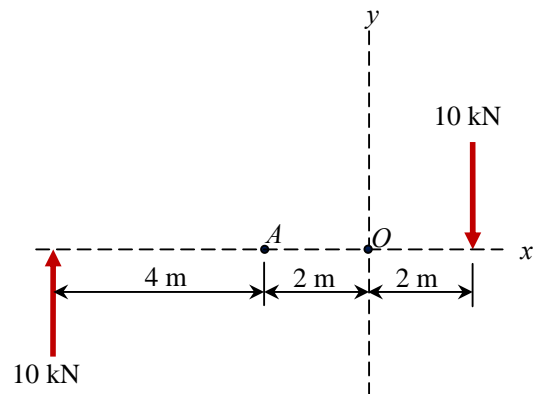
$$y_A = -\frac{M}{R_x} = -\left(\frac{-1}{-4}\right) = -0.25 \text{ m} \quad \text{Ans.}$$

**(2 marks)**

**Question # 1(c) (2 Marks)**

Compute the moment of the two 10-kN forces about the

- Point  $O$ ; and
- Point  $A$ .



**Solution**

The two 10-kN forces are forming a couple. As couple moment is independent of the moment centers, the moment produced by the two 10-kN forces about points  $O$  and  $A$  will be the same.

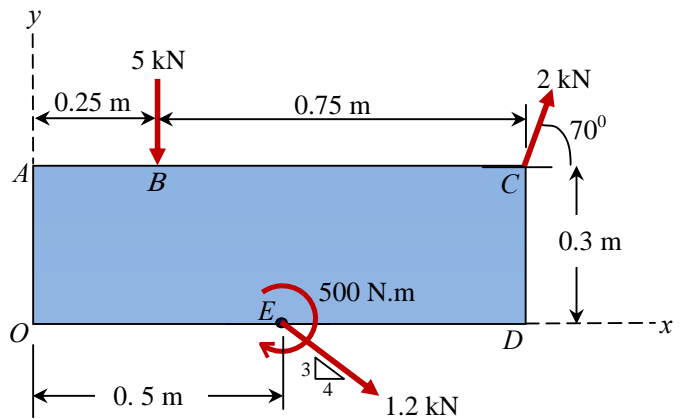
Therefore,  $M_O = M_A = Fd = -10 \times 8 = -80 \text{ kN.m (CW)}$       *Ans.*

**(2 marks)**

**Question # 2 (10 Marks)**

For the force-system shown in the figure:

- i. Replace the three forces and one couple by an equivalent force-couple system ( $R$  and  $M$ ) at point  $O$ .
- ii. Determine the direction of  $R$ .
- iii. Sketch the single resultant force  $R$  that represents the force-couple system and find its intersection with the  $x$ - and  $y$ -axes.



**Solution**

(i)

$$\rightarrow R_x = \sum F_x = 2 \cos 70^\circ + 1.2 \cos \theta = 2 \cos 70^\circ + 1.2 \times (4/5) = 1.64 \text{ kN} \rightarrow$$

(1 mark)

$$\uparrow R_y = \sum F_y = -5 + 2 \sin 70^\circ - 1.2 \sin \theta = -5 + 2 \sin 70^\circ - 1.2 \times (3/5) = -3.84 \text{ kN} \downarrow$$

(1 mark)

Therefore,  $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1.64)^2 + (-3.84)^2} = 4.18 \text{ kN}$      *Ans.*

(1 mark)

$$CCW(+M_o = -5 \times 0.25 - 2 \cos 70^\circ \times 0.3 + 2 \sin 70^\circ \times 1.0 - 1.2 \sin \theta \times 0.5 - \left(\frac{500}{1000}\right)$$

(3 marks)

$$\Rightarrow M_o = -1.25 - 0.205 + 1.879 - 1.2 \times (3/5) \times 0.5 - 0.5 = -0.44 \text{ kN.m (CW)} \quad \textit{Ans.}$$

(ii)

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-3.84}{1.64}\right) = -66.9^\circ \quad \textit{Ans.}$$

(1 mark)

(iii)

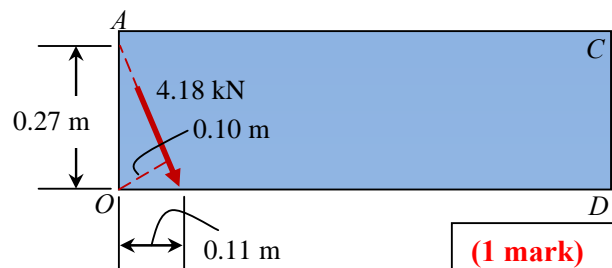
$$|M_o| = Rd \Rightarrow d = \frac{|M_o|}{R} = \frac{0.44}{4.18} = 0.10 \text{ m}$$

$$x\text{-intercept, } x = \frac{M_o}{R_y} = \frac{-0.44}{-3.84} = 0.11 \text{ m} \quad \textit{Ans.}$$

(1 mark)

$$y\text{-intercept, } y = -\frac{M_o}{R_x} = -\frac{-0.44}{1.64} = 0.27 \text{ m} \quad \textit{Ans.}$$

(1 mark)

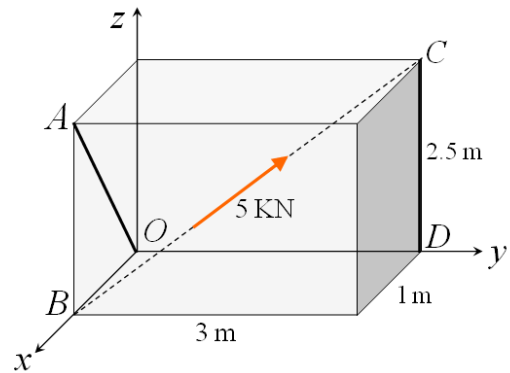


(1 mark)

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**Question # 3 (10 Marks)**

A force of 5 kN is acting along the line  $BC$  as shown in the figure. Determine the following:



- (i) The moment about point  $O$  ( $M_O$ ).
- (ii) The moment about line  $OA$  ( $M_{OA}$ ).
- (iii) The moment about line  $CD$  ( $M_{CD}$ ).

**Solution**

The coordinates of points  $O$ ,  $A$ ,  $B$  and  $C$  are:  $O(0, 0, 0)$ ;  $A(1, 0, 2.5)$ ;  $B(1, 0, 0)$  and  $C(0, 3, 2.5)$  (1 mark)

(i)  $\vec{M}_O = \vec{r}_{OB} \times \vec{F}$ , where  $\vec{r}_{OB} = \vec{i}$  and  $\vec{F} = 5\vec{u}_{BC} = 5 \left( \frac{-\vec{i} + 3\vec{j} + 2.5\vec{k}}{\sqrt{(-1)^2 + (3)^2 + (2.5)^2}} \right) = -1.24\vec{i} + 3.72\vec{j} + 3.1\vec{k}$  kN (2 marks)

Therefore,  $\vec{M}_O = \vec{r}_{OB} \times \vec{F} = \vec{i} \times (-1.24\vec{i} + 3.72\vec{j} + 3.1\vec{k}) = -3.1\vec{j} + 3.72\vec{k}$  kN.m *Ans.* (2 marks)

And the magnitude of  $\vec{M}_O$  is,  $M_O = |\vec{M}_O| = \sqrt{(-3.1)^2 + (3.72)^2} = 4.84$  kN.m *Ans.*

(ii)  $M_{OA} = \vec{M}_O \cdot \vec{u}_{OA}$ , where  $\vec{u}_{OA}$  is the unit vector along the line  $OA$ .

$\vec{u}_{OA} = \frac{\vec{r}_{OA}}{|\vec{r}_{OA}|} = \frac{\vec{i} + 2.5\vec{k}}{\sqrt{1^2 + 2.5^2}} = 0.371\vec{i} + 0.928\vec{k}$  (1 mark)

Therefore,  $M_{OA} = \vec{M}_O \cdot \vec{u}_{OA} = (-3.1\vec{j} + 3.72\vec{k}) \cdot (0.371\vec{i} + 0.928\vec{k}) = 3.45$  kN.m *Ans.* (2 marks)

The above moment can be expressed in a vector form as

$\vec{M}_{OA} = M_{OA}\vec{u}_{OA} = 3.45(0.371\vec{i} + 0.928\vec{k}) = 1.28\vec{i} + 3.20\vec{k}$  kN.m *Ans.*

(iii) Since the line of action of the force is passing through the line  $CD$ , the moment of the force about line  $CD$  will be zero. That is,  $M_{CD} = 0$  *Ans.* (2 marks)