Q1 موجودة في ملزمة المحاضرة

Q2 $X \sim \chi^2_v$

$$f(x) = \left(\frac{1}{2}\right)^{\frac{v}{2}} \frac{1}{\Gamma\frac{v}{2}} e^{-\frac{1}{2}x} x^{\frac{v}{2}-1}, \quad x>0$$

$$M_X(t) = E(e^{Xt}) = \left(\frac{1}{2}\right)^{\frac{v}{2}} \frac{1}{\Gamma\frac{v}{2}} \int_0^\infty e^{xt} e^{-\frac{1}{2}x} x^{\frac{v}{2}-1} dx$$

$$= \left(\frac{1}{2}\right)^{\frac{v}{2}} \frac{1}{\Gamma\frac{v}{2}} \int_0^\infty e^{-x\overbrace{\frac{(1-2t)}{2}}^{\beta}} x^{\overbrace{\frac{v}{2}-1}^{\alpha}} dx$$

$$\int_0^\infty e^{-\beta x} x^\alpha dx = \frac{\Gamma\alpha+1}{\beta^{\alpha+1}}$$

$$= \left(\frac{1}{2}\right)^{\frac{v}{2}} \frac{1}{\Gamma\frac{v}{2}} \frac{\Gamma\frac{v}{2}}{\left(\frac{(1-2t)}{2}\right)^{\frac{v}{2}}} = \frac{1}{(1-2t)^{\frac{v}{2}}} = (1-2t)^{-\frac{v}{2}}$$

Q3 $X_1 \sim \chi^2_{v_1}$ and $X_2 \sim \chi^2_{v_2}$ are independent

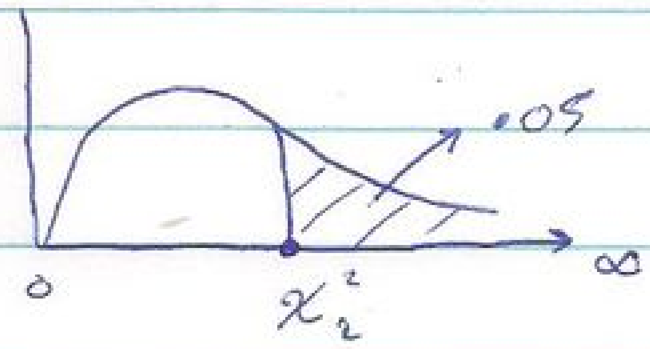
(a) $M_Z(t) = M_{X_1+X_2}(t) = M_{X_1}(t)\, M_{X_2}(t) = (1-2t)^{-\frac{v_1}{2}} (1-2t)^{-\frac{v_2}{2}} = (1-2t)^{-\frac{(v_1+v_2)}{2}}$

(b) $Z \sim \chi^2_{v_1+v_2}$

Q4 موجودة في ملزمة المحاضرة

Q5 $\quad X \sim \chi^2_{v=5}$

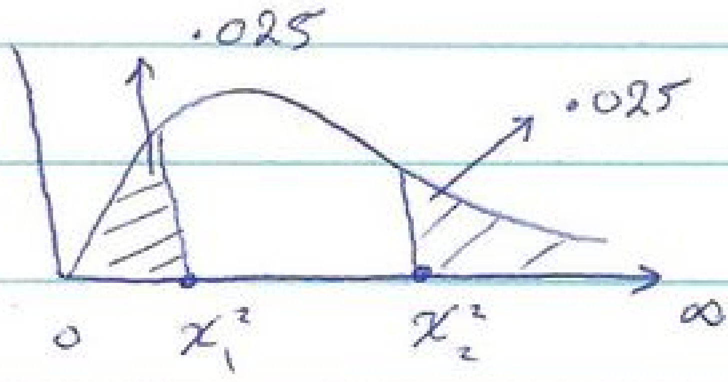
ⓐ



$P(\chi^2_v > \chi^2_2) = .05 \Rightarrow 1 - P(\chi^2_v < \chi^2_2) = .05$

$\Rightarrow P(\chi^2_v < \chi^2_2) = .95 \Rightarrow \chi^2_{v,.95} = \chi^2_2 = 11.07$

ⓑ $\frac{.05}{2} = .025$



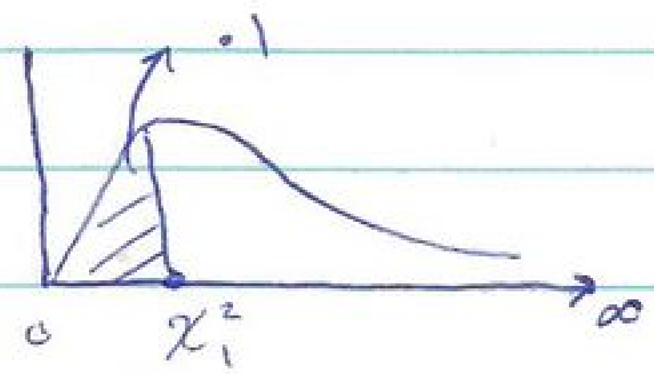
Since the distribution of χ^2_v is not symmetric, $\chi^2_1 \neq \chi^2_2$

* $P(\chi^2_v < \chi^2_1) = .025 \Rightarrow \chi^2_{v,.025} = \chi^2_1 = .831$

* $P(\chi^2_v > \chi^2_2) = .025$

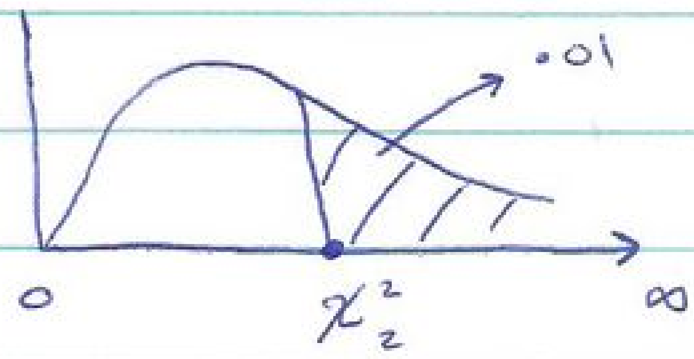
$\Rightarrow 1 - P(\chi^2_v < \chi^2_2) = .025 \Rightarrow P(\chi^2_v < \chi^2_2) = .975 \Rightarrow \chi^2_{v,.975} = \chi^2_2 = 12.83$

✓ ⓒ



$P(\chi^2_v < \chi^2_1) = .1 \Rightarrow \chi^2_{v,.1} = \chi^2_1 = 1.61$

✓ ⓓ



$P(\chi^2_v > \chi^2_2) = .01$

$\Rightarrow 1 - P(\chi^2_v < \chi^2_2) = .01$

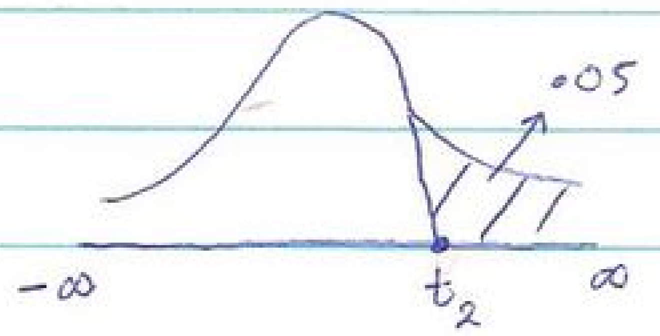
$\Rightarrow P(\chi^2_v < \chi^2_2) = .99$

$\Rightarrow \chi^2_{v,.99} = \chi^2_2 = 15.09$

Q6

$X \sim T_{v=9}$

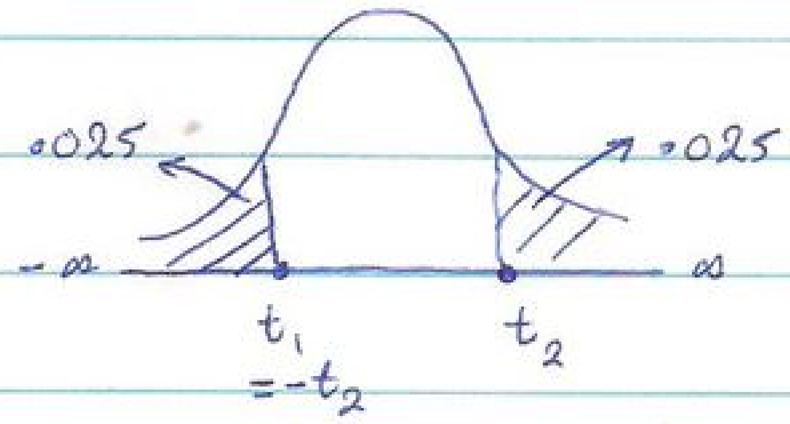
(a)



$P(T_v > t_2) = .05 \Rightarrow 1 - P(T_v < t_2) = .05$

$\Rightarrow P(T_v < t_2) = .95 \Rightarrow t_{v,.95} = t_2 = 1.833$

(b) $\frac{.05}{2} = .025$



Since the distribution is symmetry

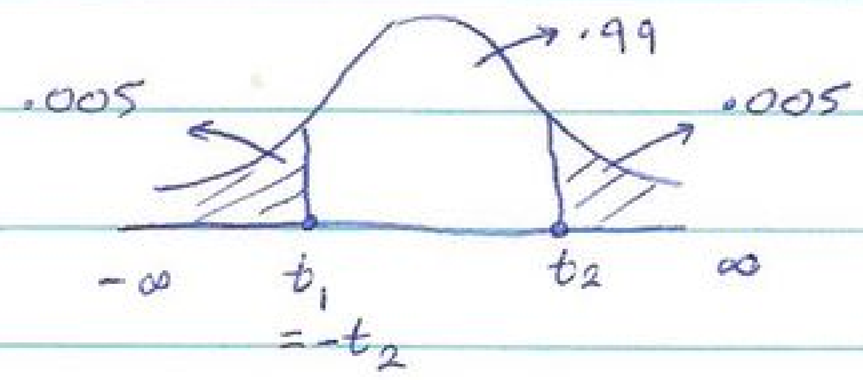
i.e $t_1 = -t_2$

$P(T_v > t_2) = .025 \Rightarrow 1 - P(T_v < t_2) = .025 \Rightarrow P(T_v < t_2) = .975$

$\Rightarrow t_{v,.975} = t_2 = 2.262$

$\therefore t_1 = -t_2 = -2.262$

(c) $1 - .99 = .01 \Rightarrow \frac{.01}{2} = .005$



Since the distribution is symmetry

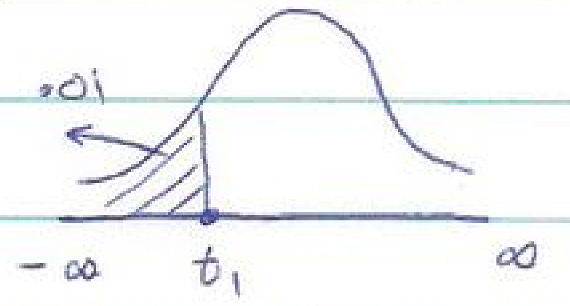
i.e $t_1 = -t_2$

$P(T_v > t_2) = .005 \Rightarrow 1 - P(T_v < t_2) = .005 \Rightarrow P(T_v < t_2) = .995$

$\Rightarrow t_{v,.995} = t_2 = 3.250$

$\therefore t_1 = -t_2 = -3.250$

(d)

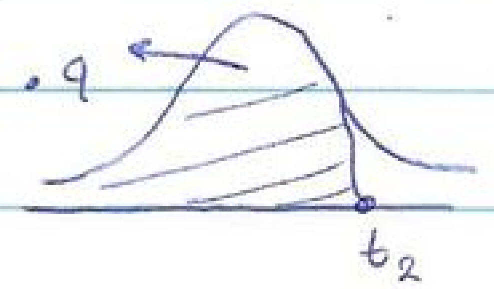


$P(T_v < t_1) = .01 \Rightarrow t_1 = t_{v,.01} = -t_{v,1-.01} = -t_{v,.99}$

$= -2.821$

(from the symmetry of T distribution)

(e)



$P(T_v < t_2) = .9 \Rightarrow t_2 = t_{v,.9} = 1.383$

Q7 $X \sim \exp(\theta = \ln(3))$ $\quad f(x) = \theta e^{-\theta x} = \ln(3) e^{-\ln(3) x}, \ x > 0$

$F(x) = 1 - e^{-\theta x} = 1 - e^{-\ln(3) x}, \ x > 0$

$P(2 \le X \le 4) \xrightarrow{\text{pdf}} = \int_2^4 f(x)\,dx = \ln(3) \int_2^4 e^{-\ln(3)x}\,dx = .0988$ (بالآلة الحاسبة)

$\xrightarrow{\text{cdf}} = P(X \le 4) - P(X \le 2) = 1 - e^{-\ln(3) \cdot 4} - (1 - e^{-\ln(3) \cdot 2}) = .0988$

Q8 $X \sim \exp(\theta = 1)$ $\quad f(x) = \theta e^{-\theta x} = e^{-x}, \ x > 0$

$F(x) = 1 - e^{-\theta x} = 1 - e^{-x}, \ x > 0$

$P(X > 2) \xrightarrow{\text{pdf}} = \int_2^\infty f(x)\,dx = \int_2^\infty e^{-x}\,dx = \frac{e^{-x}}{-1}\Big|_2^\infty = -(e^{-\infty} - e^{-2}) = e^{-2}$

$= 1 - P(X < 2) = 1 - (1 - e^{-2}) = e^{-2}$

Q9

$X \sim \exp(\theta) \Rightarrow f(x) = \theta e^{-\theta x}, \ x > 0 \ ; \ F(x) = 1 - e^{-\theta x}, \ x > 0 \ ; \ E(X) = \frac{1}{\theta}$

$P(X < E(X)) = P(X < \frac{1}{\theta}) = F(\frac{1}{\theta}) = 1 - e^{-\theta \frac{1}{\theta}} = 1 - e^{-1}$

Q10 (a) $X \sim \chi^2_v \Rightarrow M_X(t) = \left(\frac{1}{1-2t}\right)^{\frac{v}{2}}, \quad t < \frac{1}{2}$

$\therefore \frac{v}{2} = 1 \Rightarrow v = 2$

or $X \sim exp(\theta) \Rightarrow M_X(t) = \frac{\theta}{\theta - t}, \quad t < \theta$

$\therefore \frac{1}{1-2t} = \frac{\frac{1}{2}}{\frac{1}{2} - t} \Rightarrow \theta = \frac{1}{2}$

(b) $X \sim N(\mu, \sigma^2) \Rightarrow M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t}$

$\therefore \mu = 3$ and $\frac{1}{2}\sigma^2 = 2 \Rightarrow \sigma^2 = 4$

(c) X and Y are indep.

$M_{X+Y}(t) = \left(\frac{2}{2-t}\right)^3, \quad Y \sim exp(\theta = 2) \Rightarrow M_Y(t) = \frac{2}{2-t}$

$\therefore M_{X+Y}(t) = M_X(t)\, M_Y(t)$

$\Rightarrow \left(\frac{2}{2-t}\right)^3 = M_X(t)\left(\frac{2}{2-t}\right)$

$\Rightarrow M_X(t) = \frac{\left(\frac{2}{2-t}\right)^3}{\left(\frac{2}{2-t}\right)} = \left(\frac{2}{2-t}\right)^2 = \left(\frac{\beta}{\beta - t}\right)^{\alpha}$

$\therefore X \sim$ gamma $(\alpha = 2, \beta = 2)$

Q11 X and Y are indep.

$$M_{X+Y}(t) = \frac{e^{2t}-1}{2t-t^2}$$

$$X \sim exp(\theta=2) \Rightarrow M_X(t) = \frac{2}{2-t}$$

$$M_{X+Y}(t) = M_X(t)\, M_Y(t)$$

$$\Rightarrow \frac{e^{2t}-1}{2t-t^2} = \frac{2}{2-t} M_Y(t)$$

$$\Rightarrow M_Y(t) = \frac{\frac{e^{2t}-1}{t(2-t)}}{\frac{2}{2-t}} = \frac{e^{2t}-1}{t(2\cancel{-t})} \cdot \frac{2\cancel{-t}}{2} = \frac{e^{2t}-1}{2t}$$

$$= \frac{e^{2t}-e^{0}}{t(2-0)}$$

$\therefore$ X $\sim$ uniform ($a=0$, $b=2$)