King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2016–2017
Academic Year (H) 1437–1438
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Final Exam Quantitative Methods in Finance Actu. 468 (40%)

Monday, January 16, 2016 / Rabi II 18, 1438 (three hours)

Problem 1. (8 marks)

We specify below the basic elements of a financial market with T periods:

- A finite probability space $\Omega = \{\omega_1, \ldots, \omega_k\}$ with k elements.
- A probability measure P on Ω, such that $P(\omega) > 0$ for all $\omega \in \Omega$.
- A riskless asset (a saving account) $S_t^0, t \in \{0, 1, 2, \ldots T\}$ such that $S_0^0 = 1$ with a constant interest rate r.
- A d–dimensional price process S_t, $t \in \{0, 1, 2, \ldots T\}$ where $S_t = (S_t^0, S_t^1, \ldots, S_t^d)$ and S_t^i stands for the price of the asset i at time t.

Assume that $k = 3$, $d = 1$, and $T = 2$ and consider the following model

n	S_n^0	S_n^1			
		ω_1	ω_2	ω_3	ω_4
0	1	5	5	5	5
1	$1+r$	8	8	4	4
2	$(1+r)^2$	9	6	6	3

(for simplicity of computations take $r = 0$)

1. (**1 mark**) Find the sigma algebras $\mathcal{F}_0$ and $\mathcal{F}_2$.
2. (**1 mark**) Find the sigma algebra $\mathcal{F}_1$.
3. (**1 mark**) Let Q be a probability on $\Omega = \{\omega_1, \ldots, \omega_4$, Calculate $E_Q\left[S_2^1 \mid \mathcal{F}_1\right]$
4. (**1 mark**) Is this model a arbitrage free? explain your answer
5. (**1 mark**) Is this model complete? explain your answer
6. (**1 mark**) Consider a put option with strike price 7. Give the payoff of this put. Is is attainable? explain your answer
7. (**1 mark**) Find the price of this option at times 0 and 1. (+ **1 mark bonus**)
8. (**1 mark**) Find the hedging portfolio (+ **1 mark bonus**)

Solution.

1. $\mathcal{F}_0 = \{\emptyset, \Omega\}$ and $\mathcal{F}_2 = \mathcal{P}(\Omega)$ the power set of Ω
2. $\mathcal{F}_1 = \{\emptyset, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \Omega\}$.

3. We have

$$E_Q\left[S_2^1 \mid \mathcal{F}_1\right] = \begin{cases} \dfrac{9Q(\omega_1)+6Q(\omega_2)}{Q(\omega_1)+Q(\omega_2)} & \text{on} \quad \{\omega_1,\omega_2\} \\ \dfrac{6Q(\omega_3)+3Q(\omega_4)}{Q(\omega_3)+Q(\omega_4)} & \text{on} \quad \{\omega_3,\omega_4\} \end{cases}$$

4. If a RNPM $Q=(q_1,q_2,q_3,q_4)$ exists then it should satisfy

$$\text{i)}\ \ E_Q\left[S_1^1\right] = S_0^1 \ \text{ and }\ \text{ii)}\ \ E_Q\left[S_2^1 \mid \mathcal{F}_1\right] = S_1^1$$

Theses implies that

$$\text{i)} \iff 5 = 8(q_1+q_2)+4(q_3+q_4)$$

$$\text{ii)} \iff 8 = \frac{9q_1+6q_2}{q_1+q_2} \quad \text{and} \quad 4 = \frac{6q_3+3q_4}{q_3+q_4}$$

this implies that

$$\begin{cases} 5 = 8(q_1+q_2)+4(1-q_1-q_2) \\ 8\,(q_1+q_2) = 9q_1+6q_2 \\ 4\,(q_3+q_4) = 6q_3+3q_4 \end{cases} \iff \begin{cases} 4(q_1+q_2) = 1 \\ 2q_2 = q_1 \\ q_4 = 2q_3 \end{cases} \iff \begin{cases} q_1 = \dfrac{1}{6} \\ q_2 = \dfrac{1}{12} \\ q_4 = 2q_3 \end{cases}$$

which implies that $q_3 = \dfrac{1}{4}$ and $q_4 = \dfrac{1}{2}$, hence the RNPM

$$Q = \left(\frac{1}{6}, \frac{1}{12}, \frac{1}{4}, \frac{1}{2}\right)$$

therefore the model is arbitrage free since a RNPM exists.

5. The Market model is complete since the RNPM is unique.

6. The payoff of the 7–strike put is given by

$$(7-S_2)^+ = \begin{cases} 0 & \text{on} \quad \{\omega_1\} \\ 1 & \text{on} \ \{\omega_2,\omega_3\} \\ 4 & \text{on} \quad \{\omega_4\} \end{cases}$$

The put is attainable because the market is complete (any contingent claim is attainable).

7. The price or the value of the option at **time one** is given by

$$V_1 = E_Q\left[(7-S_2)^+ \mid \mathcal{F}_1\right] = \begin{cases} \dfrac{q_2}{q_1+q_2} = \dfrac{\frac{1}{12}}{\frac{1}{6}+\frac{1}{12}} = \dfrac{1}{3} & \text{on} \quad \{\omega_1,\omega_2\} \\ \dfrac{q_3+4q_4}{q_3+q_4} = \dfrac{\frac{1}{4}+4\frac{1}{2}}{\frac{1}{4}+\frac{1}{2}} = 3 & \text{on} \quad \{\omega_3,\omega_4\} \end{cases}$$

There no discounting factor because $r=0$. And the value of the option at **time zero** is given by

$$\begin{aligned} V_0 &= E_Q\left[V_1 \mid \mathcal{F}_0\right] = E_Q\left[V_1\right] = \frac{1}{3}(q_1+q_2)+3(q_3+q_4) \\ &= \frac{1}{3}\left(\frac{1}{6}+\frac{1}{12}\right)+3\left(\frac{1}{4}+\frac{1}{2}\right) = \frac{7}{3}. \end{aligned}$$

8. The hedging portfolio (α_0, Δ_0) at **time zero** should satisfies $1/0.95 = 1.0526$

$$\text{i)}\ \ V_0 = \alpha_0 + \Delta_0 S_0 \quad \text{and} \quad \text{ii)}\ \ V_1 = \alpha_0 + \Delta_0 S_1 \tag{1}$$

and the hedging portfolio (α_1, Δ_1) at **time one** should satisfies

$$\text{iii)}\ V_1 = \alpha_1 + \Delta_1 S_1 \quad \text{and} \quad \text{iv)}\ V_2 = \alpha_1 + \Delta_1 S_2 = (7 - S_2)^+ \tag{2}$$

The equation (1) leads to the system

$$\begin{cases} \frac{1}{3} = \alpha_0 + 8\Delta_0 & \text{on} \quad \{\omega_1, \omega_2\} \\ 3 = \alpha_0 + 4\Delta_0 & \text{on} \quad \{\omega_3, \omega_4\} \end{cases} \iff \left[\alpha_0 = \frac{17}{3}, \Delta_0 = -\frac{2}{3}\right]$$

and the equation (2) leads to the system

$$\begin{cases} 0 = \alpha_1^1 + 9\Delta_1^1 & \text{on} \quad \{\omega_1\} \\ 1 = \alpha_1^1 + 6\Delta_1^1 & \text{on} \quad \{\omega_2\} \end{cases} \quad \text{and} \quad \begin{matrix} 1 = \alpha_1^2 + 6\Delta_1^2 & \text{on} \quad \{\omega_3\} \\ 4 = \alpha_1^2 + 3\Delta_1^2 & \text{on} \quad \{\omega_4\} \end{matrix}$$

$$\iff \left[\Delta_1^1 = -\frac{1}{3}, \alpha_1^1 = 3\right] \quad \text{and} \quad \left[\Delta_1^2 = -1, \alpha_1^2 = 7\right].$$

Therefore

$$(\alpha_1, \Delta_1) = \begin{cases} \left(3, -\frac{1}{3}\right) & \text{on} \quad \{\omega_1, \omega_2\} \\ (7, -1) & \text{on} \quad \{\omega_3, \omega_4\}. \end{cases}$$

Problem 2. (8 marks)

Consider stock with a current price \$100 and a constant annualized volatility σ of 20%. The stock does not pay dividends. A risk–less asset is worth \$0.95 today and is worth \$1 in one year maturity. Consider also European and American put options on the stock with a maturity of **two years** and a strike price of \$110.

1. **(1 mark)** Using the approach discussed in the lectures, construct a **two–step** binomial tree to approximate the stock price dynamics, with each step being **one year**. Build the stock price at each **node** at **one** and **two** years.
2. **(1 mark)** Compute the risk–neutral probability measure.
3. **(1 mark)** Give the binomial tree of the European put option.
4. **(1 mark)** Based on the binomial tree, find the current value of the European put option.
5. **(1 mark)** Compute the delta of the European put option for the first and the second periods.
6. **(1 mark)** Give the binomial tree of the American put option.
7. **(1 mark)** Based on the binomial tree, find the current value of the American put option.
8. **(1 mark)** Compute the delta of the American put option for the first and the second periods.

Solution:

1. We have $T = 2$, then $h = 1$, hence $u = e^{\sigma\sqrt{h}}$ with $\sigma = 0.20$, thus $u = e^{0.20\sqrt{1}} = 1.2214$ and $d = \frac{1}{u} = \frac{1}{1.2214} = 0.8187$.

2-step 1-year stock tree			
	t=0	t=1	t=2
			149,1825
		122,1403	
Currency tree	100,0000		100,0000
		81,8731	
			67,0320

2. The risk–neutral probability measure is given by

$$q = \frac{a - d}{u - d} \quad \text{where} \quad a = e^{rh}$$

We have the risk–free rate is

$$r = \frac{1 - 0.95}{0.95} = 0.0526$$

Therefore

$$a = e^{0.0526} = 1.054 \quad \text{and} \quad q = \frac{1.054 - 0.8187}{1.2214 - 0.8187} = 0.5844$$

3. The binomial tree of the European put option is given by

2-step 1-year European put option			
	t=0	t=1	t=2
			0,0000
		3,9432	
Option tree	11,0532		10,0000
		22,4872	
			42,9680

4. The price of the European put option is then from the tree 11.0532.

5. The delta of the European put option at times 0 and 1 is given by the following tree:

The tree of delta of the European put option

t=0	t=1
	-0,2033
-0,4605	
	-1,0000

6. The binomial tree of the American put option is given by:

2-step 2-year American put option

	t=0	t=1	t=2
			0,0000
		3,9432	
Option tree	13,2771		10,0000
		28,1269	
			42,9680

7. The price of the American put option is then from the tree 13.2771.

8. The delta of the American put option at times 0 and 1 is given by the following tree:

The tree of delta of the American put option	
t=0	t=1
	-0,2033
-0,6006	
	-1,0000

Problem 3. (8 marks)

An XYZ asset is currently priced at \$700 per share and an analyst predicts that the stock will hit 1000 at some point in the next $T = 1.5$ years. Assume that the stock price S_t of XYZ is given by the Black–Scholes formula. The stock pays no dividends, and the risk–free interest rate is $r = 6\%$. Take the rate of return of the stock price to be $\mu = 10\%$ and the volatility to be $\sigma = 30\%$.

1. **(1 mark)** Give the expression of S_T under the historical probability
2. **(1 mark)** Give the distribution of $\ln(\frac{S_T}{S_0})$.
3. **(1 mark)** Let $\mathbf{N}(\cdot)$ be the c.d.f. of the S.N.D. $\mathcal{N}(0,1)$, we know that $\mathbf{N}(x) + \mathbf{N}(-x) = 1$ for all x. Calculate the probability that $S_{1.5} > 1000$ in this model assuming that $\mathbf{N}(0.7463) = 0.7722$.
4. **(1 mark)** To take advantage of the expected rise in XYZ stock, you decide to buy a call option with strike 1000 on one share of XYZ. The option expires in 1.5 years and implied volatility is 30%.
   What are the input parameters for the Black–Scholes formula to value this option?
5. **(1 mark)** What are their values?
6. **(1 mark)** Given $\mathbf{N}(-0.5421) = 0.2939$ and $\mathbf{N}(-0.9095) = 0.1815$. What is the Black–Scholes value of your call option at the time of purchase?
7. **(1 mark)** Find the replicating portfolio from the side of the seller of this option today?
8. **(1 mark)** Suppose that the **6 months** price of XYZ stock is 1000. Given $\mathbf{N}(0.35) = 0.6368$ and $\mathbf{N}(0.05) = 0.52$ what would the value of your option be then at this time?

Solution:

1. Under the Black–Scholes model we have $S_T = S_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma W_T}$ where $(W_t)_{t \geq 0}$ is a standard Brownian motion.
2. We have $\ln(\frac{S_T}{S_0}) = (\mu - \frac{\sigma^2}{2})T + \sigma W_T$ then $\ln(\frac{S_T}{S_0})$ follows a Normal distribution $\mathcal{N}((\mu - \frac{\sigma^2}{2})T, \sigma^2 T)$ which is for $T = 1.5$ $\mathcal{N}(0.0825, 0.135)$ because $(0.1 - \frac{0.3^2}{2})1.5 = 0.0825$ and $0.3^2 1.5 = 0.135$.

3. We can write for $\ln(\frac{10}{7}) = 0.3567$

$$\begin{aligned} P\left(S_{1.5} > 1000\right) &= P\left(\ln\left(\frac{S_{1.5}}{S_0}\right) > \ln\left(\frac{10}{7}\right)\right) = P\left(0.0825 + 0.3W_{1.5} > 0.3567\right) \\ &= P\left(0.3W_{1.5} > 0.2742\right) = P\left(0.3\sqrt{1.5}W_1 > 0.2742\right) \\ &= 1 - P\left(W_1 \leq \frac{0.2742}{0.3\sqrt{1.5}}\right) = 1 - P\left(W_1 \leq 0.7463\right) = 1 - 0.7722 = 0.2278 \end{aligned}$$

4. The input parameters for the Black–Scholes formula to value this option are: initial price S_0, strike price K, Maturity T, risk–free rate r, the volatility σ, the dividend q.

5. The values of the inputs are $S_0 = \$700$, $K = 1000$, $T = 1.5$, $r = 0.06$, $\sigma = 0.3$ and $q = 0$.

6. The call option price is given by the Black–Scholes formula

$$C_0 = S_0\mathbf{N}(d_+) - e^{-rT}K\mathbf{N}(d_-)$$

where

$$d_+ = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \text{ you can also use } d_- = d_+ - \sigma\sqrt{T}$$

Therefore with our data we have

$$d_+ = \frac{\ln(\frac{7}{10}) + (0.06 + \frac{0.3^2}{2})1.5}{0.3\sqrt{1.5}} = -0.5421 \text{ and } d_- = -0.5421 - 0.3\sqrt{1.5} = -0.9095$$

and the price of the call option is given by

$$\begin{aligned} 700\mathbf{N}(-0.5421) - e^{-0.06\times 1.5}1000\mathbf{N}(-0.9095) &= 700 \times 0.2939 - e^{-0.06\times 1.5}1000 \times 0.1815 \\ &= 39.8 \simeq 40. \end{aligned}$$

7. The seller has a short position on this option, hence this delta is $-\mathbf{N}(d_+) \simeq -0.3$ to hedge this position the seller should buy $\frac{3}{10}$ shares of the stock which costs $\frac{3}{10} \times 700 = 210$, thus he/she should borrow $210 - 40 = 170$ dollars at 6%.

8. If $S_{6\ months} = S_{0.5} = 1000$ then

$$d_+ = \frac{(0.06 + \frac{0.3^2}{2})}{0.3} = 0.35 \text{ and } d_- = 0.35 - 0.3 = 0.05$$

and the price of the call option is given by

$$\begin{aligned} 1000\mathbf{N}(0.35) - e^{-0.06}1000\mathbf{N}(0.05) &= 1000\left(0.6368 - e^{-0.06}0.52\right) \\ &= 147.08. \end{aligned}$$

Problem 4. (8 marks)

Assume that the market–maker is trading options on an underlying asset with the price process $(S_t)_{t\geq 0}$ with initial price $S_0 = 40$, a volatility of 25% and the risk–free rate $r = 5\%$. Assume that the asset pays a dividend yield $q = 2\%$. The market–maker sells a 42–strike put on $(S_t)_{t\geq 0}$ in three months from now.

1. **(1 mark)** Give the formula of the Black–Scholes price of the above put ?

2. **(1 mark)** Given $\mathbf{N}(0.3928) = 0.6527$ and $\mathbf{N}(0.2678) = 0.6056$, find numerically the put price

3. **(1 mark)** Give the formula of the delta of the above put?

4. **(1 mark)** Find a numerical value of the delta

5. **(1 mark)** The market–maker sells 100 shares of the above put and decides to delta–hedge this position. What investment is required to do so?

6. **(1 mark)** If the next month price of the stock is 42. Find the one month price of the option assuming that $\mathbf{N}(0.002) = 0.5$ and $\mathbf{N}(-0.1) = 0.46$.

7. **(1 mark)** Find the one month delta of 100 shares of the put.

8. **(1 mark)** Find the hedging strategy of the above position.

Solution:

1. The Black–Scholes price of the above put

$$P_0 = Ke^{-rT}\mathbf{N}(-d_-) - S_0e^{-qT}\mathbf{N}(-d_+),$$

where

$$d_+ = \frac{\ln(\frac{S_0}{K}) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad \text{and} \quad d_- = d_+ - \sigma\sqrt{T}.$$

2. We have $S_0 = 40$, $K = 42$, $r = 0.05$, $q = 0.02$, $\sigma = 0.25$, $T = \frac{3}{12} = \frac{1}{4}$, then we have

$$d_+ = \frac{\ln(\frac{40}{42}) + (0.05 - 0.02 + \frac{0.25^2}{2})\frac{1}{4}}{0.25\sqrt{\frac{1}{4}}} \simeq -0.2678 \quad \text{and} \quad d_- = -0.2678 - 0.25\frac{1}{2} \simeq -0.3928$$

The value of one put option is

$$\begin{aligned} P_0 &= 42e^{-0.05\times0.25}\mathbf{N}(0.3928) - 40e^{-0.02\times0.25}\mathbf{N}(0.2678) \\ &= 42e^{-0.05\times0.25}0.6527 - 40e^{-0.02\times0.25} \times 0.6056 \simeq 2.97 \end{aligned}$$

3. Using the Black Scholes formula for dividend–paying stock the delta of the put is: $\Delta_{put} = e^{-qT}(\mathbf{N}(d_+) - 1) = -e^{-qT}\mathbf{N}(-d_+)$

4. The numerical value of the delta is $\Delta_{put} = e^{-0.02\times0.25}(\mathbf{N}(d_+) - 1) = -e^{-0.02\times0.25}\mathbf{N}(-d_+) = -e^{-0.02\times0.25} \times 0.6056 \simeq -0.60$.

5. If the market–maker sells 100 of the puts the corresponding values of the portfolio is $-100P_0$, to delta hedge this position he/she should take position on the stock say with some quantity β, hence the value of his portfolio will be $-100P_0 + \beta S_0$, so the delta of this portfolio is zero when $\beta = 100\Delta_{put} = -60$, so market–maker will have to short 60 shares of the stock at time zero. This implies our delta hedge will require **shorting** 60 shares. Doing so, he/she receives $60 \times \$40 = \$2400$. There will now be interest earned since we are receiving both the option premium \$297 as well as the \$2400 on the short sale. So, he/she invests in the money market the total amount $\$297 + \$2400 = \$2697$.

6. If the price of the stock in the next month is 42 then the price of the option in this time (two months before expiration). We have $S_1 = 42$, $K = 42$, $r = 0.05$, $q = 0.02$, $\sigma = 0.25$, the time to maturity is $\frac{2}{12}$, then we have

$$d_+ = \frac{\ln(\frac{42}{42}) + (0.05 - 0.02 + \frac{0.25^2}{2})\frac{1}{6}}{0.25\sqrt{\frac{1}{6}}} \simeq 0.1 \text{ and } d_- = 0.1 - 0.25\sqrt{\frac{1}{6}} \simeq -0.002$$

The value of one put option is

$$\begin{aligned} P_1 &= 42e^{-0.05\times\frac{1}{6}}\mathbf{N}(0.002) - 42e^{-0.02\times\frac{1}{6}}\mathbf{N}(-0.1) \\ &= 42\left(e^{-0.05\times\frac{1}{6}} \times 0.5 - e^{-0.02\times\frac{1}{6}} \times 0.46\right) \simeq 1.56 \end{aligned}$$

7. Using the Black Scholes formula for dividend–paying stock the delta of the put is: $\Delta_{put} = e^{-0.02\times\frac{1}{6}}(\mathbf{N}(d_+) - 1) = -e^{-0.02\times\frac{1}{6}}\mathbf{N}(-d_+) = -e^{-0.02\times\frac{1}{6}} \times 0.4602 \simeq -0.46$. Therefore the delta of shorting 100 of the puts is the delta of $-100P_1$ which is equal to $-100\Delta_{put} = 46$ at time one.

8. Similarly to the question 4 to delta hedge this position he/she should take position on the stock say with some quantity β, hence the value of his portfolio will be $-100P_1 + \beta S_1$, so the delta of this portfolio is zero when $\beta = 100\Delta_{put} = -46$, so market–maker will have to short 46 shares of the stock at time one (one month). This implies our delta hedge will require **shorting** 46 shares. Doing so, he/she receives $46 \times 42 = 1932 = \$2400$. There will now be interest earned since he/she receives both the option premium \$156 as well as the \$1932 on the short sale. So, he/she invests in the money market the total amount $\$156 + \$1932 = \$2088$.

Problem 5. (8 marks)

A financial institution has the following portfolio of over–the–counter options on **sterling (the underlying)**

Type of option	Position	Delta of option	Gamma of option	Vega of option
Call of type 1	−1000	0.5	2.2	1.8
Call of type 2	−500	0.8	0.6	0.2
Put of type 3	−2000	−0.4	1.3	0.7
Call of type 4	−500	0.7	1.8	1.4

The sign (−) means the position is short.
A **traded option** (say of type 5) is available in the market with a **delta** of 0.6, a **gamma** of 1.5 and a **vega** of 0.8.

Denote by Π_1 the value of the above portfolio.

1. **(1 mark)** Calculate the Delta of Π_1.

2. **(1 mark)** Calculate the Gamma of Π_1 .

3. **(1 mark)** Consider a new portfolio Π_2 composed on Π_1 and β position in the **traded option**. Find β in such away that **new portfolio** Π_2 is gamma–neutral.

4. **(1 mark)** What position in the **traded option** would make the **new portfolio** Π_2 gamma–neutral?

5. **(1 mark)** Consider a new portfolio Π_3 composed on Π_2 and δ position in the **sterling (the stock)**. Find δ in such away that **new portfolio** Π_3 is delta–neutral.

6. **(1 mark)** What position in the **traded option** and in **sterling** (the stock) would make the **new portfolio** Π_3 delta–neutral and gamma–neutral?
7. **(1 mark)** Calculate the Vega of this portfolio.
8. **(1 mark)** What position in the **traded option** and in **sterling** (the stock) would make the **new portfolio both** vega–neutral and delta–neutral? **(+1 mark bonus).**

Solution:

1. The delta of the portfolio is

$$-1000 \times 0.5 - 500 \times 0.8 - 2000 \times (-0.4) - 500 \times 0.7 = -450$$

2. The gamma of the portfolio is

$$-1000 \times 2.2 - 500 \times 0.6 - 2000 \times 1.3 - 500 \times 1.8 = -6000$$

3. We have $\Pi_2 = \Pi_1 + \beta(\text{options of type 5})$, hence

$$\Gamma(\Pi_2) = \Gamma(\Pi_1) + \beta\Gamma(\text{traded option of type 5}) \text{ should be zero}$$

that is

$$0 = -6000 + \beta \times 1.5 \Longleftrightarrow \beta = \frac{6000}{1.5} = 4000$$

4. A long position in 4000 traded option of type 5 will give a gamma–neutral portfolio since the long position has a gamma of

$$4000 \times 1.5 = 6000$$

then $\Pi_2 = \Pi_1 + 4000$ (traded option of type 5)

5. We have $\Pi_3 = \Pi_1 + 4000$ (traded option of type 5) $+\delta S$ where $S =$(sterling), hence

$$\Delta(\Pi_3) = \Delta(\Pi_1) + 4000\Delta(\text{traded option of type 5}) + \delta \text{ should be zero}$$

that is

$$0 = -450 + 4000 \times 0.6 + \delta \Longleftrightarrow \delta = -1950$$

6. Hence, in addition to the 4000 traded options, a **short position** of 1950 in **sterling** is necessary so that the portfolio is both **gamma and delta neutral**.

7. The vega of the portfolio is

$$-1000 \times 1.8 - 500 \times 0.2 - 2000 \times 0.7 - 500 \times 1.4 = -4000$$

8. A long position in 5000 **traded options** will give a vega–neutral portfolio since the long position has a vega of

$$5000 \times 0.8 = 4000$$

The delta of the whole portfolio (**including traded options**) is then

$$5000 \times 0.6 - 450 = 2550.$$

Hence, in addition to the 5000 traded options, a **short position** of 2550 in **sterling** is necessary so that the portfolio is both **vega and delta neutral**.