
COMBINATIONS

Q1. A man wants to paint his house in 3 colors. If he can choose 3 colors out of 6 colors, how many different color settings can he make? 6C3

(A) 216 (B) 20 (C) 18 (D) 120 (E) 110

Q2. The number of ways in which we can select two students among a group of 5 students is 6C5

(A) 120 (B) 10 (C) 60 (D) 20 (E) 110

Q3. The number of ways in which we can select a president and a secretary among a group of 5 students is 5P2

(A) 120 (B) 10 (C) 60 (D) 20 (E) 110

Q4. If an experiment consists of throwing a die and then drawing a letter at random from the English alphabet, how many points are there in the sample space? 6 x 26=156

Q5. If a multiple-choice test consists of 5 questions, each with 4 possible answers of which only 1 is correct,

(a) in how many different ways can a student check off one answer to each question? $4^5=1024$

(b) in how many ways can a student check off one answer to each question and get all the answers wrong? $3^5=243$

Q6. How many lunches consisting of a soup, sandwich, dessert and a drink are possible if we can select from 4 soups, 3 kinds of sandwiches, 5 desserts and 4 drinks? ((4) (3) (5) (4) =240 different ways to choose lunch)

PROBABILITY, CONDITIONAL PROBABILITY, AND INDEPENDENCE

Q1. Consider the experiment of flipping a balanced coin three times independently.

(a) The number of points in the sample space is 2^3

(A) 2 (B) 6 (C) 8 (D) 3 (E) 9

(b) The probability of getting exactly two heads is P(A)=3/8

(A) 0.125 (B) 0.375 (C) 0.667 (D) 0.333 (E) 0.451

(c) The events 'exactly two heads' and 'exactly three heads' are P(A∩B)=0

(A) Independent (B) disjoint (C) equally likely (D) identical (E) None

(d) The events 'the first coin is head' and 'the second and the third coins are tails' are P(A)P(B)=P(A∩B)

(A) Independent (B) disjoint (C) equally likely (D) identical (E) None

Q2. Suppose that a fair die is thrown twice independently, then

1. the probability that the sum of numbers of the two dice is less than or equal to 4 is; P(A)=6/36
   (A) 0.1667 (B) 0.6667 (C) 0.8333 (D) 0.1389
2. the probability that at least one of the die shows 4 is; P(A)=11/36
   (A) 0.6667 (B) 0.3056 (C) 0.8333 (D) 0.1389
3. the probability that one die shows one and the sum of the two dice is four is; P(A)=2/36
   (A) 0.0556 (B) 0.6667 (C) 0.3056 (D) 0.1389
4. the event A={the sum of two dice is 4} and the event B={exactly one die shows two} are,
   (A) Independent (B) Dependent (C) Joint (D) None of these. P(A)P(B)≠P(A∩B)

Q3. Assume that $P(A)=0.3$, $P(B)=0.4$, $P(A\cap B\cap C)=0.03$, and $P(\overline{A\cap B})=0.88$, then

1. the events A and B are, P(A)P(B)=P(A∩B)
   (A) Independent (B) Dependent (C) Disjoint (D) None of these.
2. $P(C|A\cap B)$ is equal to, P(A∩B∩C)/P(A∩B)
   (A) 0.65 (B) 0.25 (C) 0. 35 (D) 0.14

Q4. If the probability that it will rain tomorrow is 0.23, then the probability that it will not rain tomorrow is:
(A) −0.23 (B) 0.77 (C) −0.77 (D) 0.23

Q5. The probability that a factory will open a branch in Riyadh is 0.7, the probability that it will open a branch in Jeddah is 0.4, and the probability that it will open a branch in either Riyadh or Jeddah or both is 0.8. Then, the probability that it will open a branch:

1) in both cities is: P(A∩B)=P(A)+P(B)-P(AUB)=0.4+0.7-0.8
(A) 0.1 (B) 0.9 (C) 0.3 (D) 0.8

2) in neither city is:1-P(AUB)=1-0.8
(A) 0.4 (B) 0.7 (C) 0.3 (D) 0.2

Q6. 200 adults are classified according to sex and their level of education in the following table:

Sex / Education	Male (M)	Female (F)
Elementary (E)	28	50
Secondary (S)	38	45
College (C)	22	17

If a person is selected at random from this group, then:

1) the probability that he is a male is:P(M)=(28+38+22)/(200)
(A) 0.3182 (B) 0.44 (C) 0.28 (D) 78

2) The probability that the person is male given that the person has a secondary education is: P(M|S)=P(M∩S)/P(S)=38/(38+45)
(A) 0.4318 (B) 0.4578 (C) 0.19 (D) 0.44

3) The probability that the person does not have a college degree given that the person is a female is: P(C′|F)=(45+50)/(50+45+17)
(A) 0.8482 (B) 0.1518 (C) 0.475 (D) 0.085

4) Are the events M and E independent? Why? [P(M)=0.44 ≠ P(M|E)=0.359 ⇒ dependent]
or P(M∩E)≠P(M)P(E)

Q7. 1000 individuals are classified below by sex and smoking habit.

		SEX	
		Male (M)	Female (F)
SMOKING HABIT	Daily (D)	300	50
	Occasionally (O)	200	50
	Not at all (N)	100	300

A person is selected randomly from this group.

1. Find the probability that the person is female. [P(F)=0.4] 400/1000
2. Find the probability that the person is female and smokes daily. [P(F∩D)=0.05] 50/1000
3. Find the probability that the person is female, given that the person smokes daily. [P(F|D)=0.1429] 50/350
4. Are the events F and D independent? Why? [P(F)=0.4 ≠ P(F|D)=0.1429 ⇒ dependent]

Q8. Two engines operate independently, if the probability that an engine will start is 0.4, and the probability that the other engine will start is 0.6, then the probability that both will start is: 0.4*0.6
(A) 1 (B) 0.24 (C) 0.2 (D) 0.5

Q9. If $P(B) = 0.3$ and $P(A|B) = 0.4$, then $P(A \cap B)$ equals to;

(A) 0.67 (B) 0.12 (C) 0.75 (D) 0.3

Q10. The probability that a computer system has an electrical failure is 0.15, and the probability that it has a virus is 0.25, and the probability that it has both problems is 0.10, then the probability that the computer system has the electrical failure or the virus is: 0.15+0.25-0.1

(A) 1.15 (B) 0.2 (C) 0.15 (D) 0.30

Q11. If $P(A_1)=0.4$, $P(A_1 \cap A_2)=0.2$, and $P(A_3|A_1 \cap A_2)=0.75$, then

(1) $P(A_2|A_1)$ equals to

(A) 0.00 (B) 0.20 (C) 0.08 (D) 0.50

(2) $P(A_1 \cap A_2 \cap A_3)$ equals to

(A) 0.06 (B) 0.35 (C) 0.15 (D) 0.08

Q12. If $P(A)=0.9$, $P(B)=0.6$, and $P(A \cap B)=0.5$, then:

(1) $P(A \cap B^C)$ equals to

(A) 0.4 (B) 0.1 (C) 0.5 (D) 0.3

(2) $P(A^C \cap B^C)$ equals to

(A) 0.2 (B) 0.6 (C) 0.0 (D) 0.5

(3) $P(B|A)$ equals to

(A) 0.5556 (B) 0.8333 (C) 0.6000 (D) 0.0

(4) The events A and B are $P(A \cap B) \neq 0$

(A) independent (B) disjoint (C) joint (D) none

(5) The events A and B are $P(A \cap B) \neq P(A)P(B)$

(A) disjoint (B) dependent (C) independent (D) none

Q13. A total of 36 members of a club play tennis, 28 play squash, 18 play badminton, 22 play both tennis and squash, 12 play both tennis and badminton, 9 play both squash and badminton, 4 play all 3. What is the probability that at least one a member of this club plays at least one sport. Assuming that the total number of members in the club is 50. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Q14. Suppose we have a fuse box containing 20 fuses of which 5 are defective D and 15 are non-defective N. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective.

$P(A)=5/20=0.25$ $P(B|A)=4/19=0.21$ $P(A \cap B)=P(A)*P(B|A)=0.25*0.21=0.0525$

Q15. Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Fined $P(A_1 \cap A_2 \cap A_3)$, where the events A_1, A_2, and A_3 are defined as follows:

A_1 = {the 1-st card is a red ace}

A_2 = {the 2-nd card is a 10 or a jack}

A_3 = {the 3-rd card is a number greater than 3 but less than 7}

$N(A1)=2$

$N(A2)=2*4=8$

$N(A3)=3*4=12$

$P(A1 \cap A2 \cap A3)=(2/52)(8/51)(12/50)=0.001$

Q16. Find the errors in each of the following statements:

(a) The probability that it will rain tomorrow is 0.40, and the probability that it will not rain tomorrow is 0.52. $P(A)+P(A')$ must equal 1. The error : $0.40+0.52=0.92 \neq 1$

(b) On a single draw from a deck of playing cards, the probability of selecting a heart is 1/4, the probability of selecting a black card is 1/2, and the probability of selecting both a heart and a black card is 1/8. The event of selecting both a heart and a black card is disjoint ; so the probability equals 0

BAYES RULE

Q1. Two machines A and B make 80% and 20%, respectively, of the products in a certain factory. It is known that 5% and 10% of the products made by each machine, respectively, are defective. A finished product is randomly selected.

(a) Find the probability that the product is defective. [P(D)=0.06] P(D)=P(A)P(D|A)+P(B)P(D|B)=0.8*0.05+0.2*0.1=0.06

(b) If the product were found to be defective, what is the probability that it was made by machine B. [P(B|D)=0.3333] P(B|D)=P(B∩D)/P(D)=0.02/0.06=0.3333

Q2. Dates' factory has three assembly lines, A, B, and C. Suppose that the assembly lines A, B, and C account for 50%, 30%, and 20% of the total product of the factory. Quality control records show that 4% of the dates packed by line A, 6% of the dates packed by line B, and 12% of the dates packed by line C are improperly sealed. If a pack is randomly selected, then:

(a) the probability that the pack is from line B and it is improperly sealed is

(A) 0.018 (B) 0.30 (C) 0.06 (D) 0.36 (E) 0.53

(b) the probability that the pack is improperly sealed is

(A) 0.62 (B) 0.022 (C) 0.062 (D) 0.22 (E) 0.25

(c) if it is found that the pack is improperly sealed, what is the probability that it is from line B?

(A) 0.0623 (B) 0.0223 (C) 0.6203 (D) 0.2203 (E) 0.2903

Q3. Two brothers, Mohammad and Ahmad own and operate a small restaurant. Mohammad washes 50% of the dishes and Ahmad washes 50% of the dishes. When Mohammad washes a dish, he might break it with probability 0.40. On the other hand, when Ahmad washes a dish, he might break it with probability 0.10. Then,

(a) the probability that a dish will be broken during washing is:

(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5

(b) If a broken dish was found in the washing machine, the probability that it was washed by Mohammad is:

(A) 0.667 (B) 0.25 (C) 0.8 (D) 0.5