King Saud University: Mathematics Department Math-254
First Semester 1442 H Midterm Examination
Maximum Marks = 30 Time: 120 mins.

Name of the Student:————————————— **I.D. No.** ————————————-

Name of the Teacher:————————————— **Section No.** —————————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 1 for each one ($1 \times 15 = 22.5$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15
a,b,c,d					

Quest. No.	Marks Obtained	Marks for Question
Q. 1 to Q. 15		22.5
Q. 16		3.5
Q. 17		4
Total		30

Question 1: The first approximation of the square root of 19 using a quadratic convergent method with $x_0 = 5$ is:

(a) 4.4 **(b)** 4.44 **(c)** 4.36 **(d)** None of these

Question 2: The first approximation of $x = \cos(x)$ using Secant method for $x_0 = 0.5$ and $x_1 = \pi/4$ is:

(a) 0.7853 **(b)** 1.5544 **(c)** 0.5544 **(d)** None of these

Question 3: The order of convergence of $x_{n+1} = 2x_n^2 + \dfrac{4}{x_n} - 5,\ n \geq 0$, to $\alpha = 1$ is:

(a) Quadratic (b) Linear (c) Cubic (d) None of these

Question 4: The iterative scheme $x_{n+1} = 2 + (3+a)x_n - ax_n^3,\ n \geq 0$ converges at least quadratically to the root $\alpha = -1$ if a is:

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) None of these

Question 5: The first approximation of the solution of the system of nonlinear equations $xy - 1 = 0$ and $xy^2 - 1 = 0$ using Newton's method with initial approximation $(x_0, y_0) = (-1, 1)$ is:

(a) $(x_1, y_1) = (1, 1)$ (b) $(x_1, y_1) = (-0.5, 0.5)$ (c) $(x_1, y_1) = (0, -0.5)$ (d) None of these

Question 6: The Jacobian matrix of the nonlinear system of the equations $x^3 + y = 1$, $-x + y^3 = -1$ at $x_0 = 0.5 = y_0$ is:

(a) $\begin{pmatrix} 0.75 & 1 \\ -1 & 0.75 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 0.75 \\ 0.75 & 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} -1 & 0.75 \\ 0.75 & 1 \end{pmatrix}$ **(d)** None of these

Question 7: Let $A = \begin{pmatrix} 1 & \alpha \\ \alpha & 4 \end{pmatrix}$, and $\mathbf{b} = [1,\ 2]^t$. If $\alpha = -2$, then using the simple Gaussian elimination the system $A\mathbf{x} = \mathbf{b}$ is:

(a) Inconsistent **(b)** has a unique solution **(c)** has many solutions **(d)** None of these

Question 8: Let $A = \begin{pmatrix} 1 & \alpha^2 \\ \alpha & 1 \end{pmatrix}$, $\mathbf{b} = [1,\ 1]^t$ and s is any real number. If $\alpha = 1$, then using the simple Gaussian elimination the solution of the system $A\mathbf{x} = \mathbf{b}$ is:

(a) $[1 - s,\ s]^t$ **(b)** $[1 + s,\ s]^t$ **(c)** $[s,\ 1 + s]^t$ **(d)** None of these

Question 9: If the matrix $A = \begin{pmatrix} 3 & 1 \\ 6 & 1 \end{pmatrix}$ is factored as LU using Doollitle's method, where L is a lower triangular matrix, and U is an upper triangular matrix, then the solution of the system $L\mathbf{y} = [-1,\ 0]^t$ is:

(a) $[-1,\ 2]^t$ **(b)** $[-1,\ -2]^t$ **(c)** $[-1,\ 6]^t$ **(d)** None of these

Question 10: If the matrix $A = \begin{pmatrix} 2 & 4 \\ -3 & 2 \end{pmatrix}$ is factored as LU using Crouts method, where L is a lower triangular matrix, and U is an upper triangular matrix, then the matrix L is:

(a) $\begin{pmatrix} 2 & 0 \\ -3 & 8 \end{pmatrix}$ **(b)** $\begin{pmatrix} 2 & 0 \\ 3 & 8 \end{pmatrix}$ **(c)** $\begin{pmatrix} 8 & 0 \\ -3 & 2 \end{pmatrix}$ **(d)** None of these

Question 11: If the matrix $A = \begin{pmatrix} 2 & -1 \\ 2 & -2 \end{pmatrix}$ is factored as LU using Doolittle's method, where L is a lower triangular matrix, and U is an upper triangular matrix, then U^{-1} is:

(a) $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & -1 \end{pmatrix}$ **(b)** $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$ **(c)** $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 \end{pmatrix}$ **(d)** None of these

Question 12: The second approximation for solving linear system $AX = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ using Jacobi iterative method where $A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ and $X^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is :

(a) $X^{(2)} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ **(b)** $X^{(2)} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **(c)** $X^{(2)} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ **(d)** None of these

Question 13: The norm $\|T_J\|_\infty$ of the Jacobi matrix T_J of the linear system $AX = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ where $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 3 & -1 \\ -1 & 3 & 4 \end{pmatrix}$ is :

(a) $\frac{3}{2}$ **(b)** $\frac{2}{3}$ **(c)** 1 **(d)** None of these

Question 14: The number of iterations needed to achieve accuracy 10^{-4} using Gauss-Seidel iterative method if $\|T_G\| = \frac{1}{3}$, $X^{(0)} = (1, 0, -1)^T$ and $X^{(1)} = (1.2, 2.3, 3.1)^T$ is :

(a) 11 **(b)** 9 **(c)** 8 **(d)** None of these

Question 15: The error bound of $\|x - x^{(4)}\|_\infty$ using Jacobi iterative method with $x^{(0)} = (0, 0)^T$ for solving linear system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is:

(a) $\frac{1}{8}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{2}{5}$ **(d)** None of these

Question 16: Find the rate of convergence of the Newton's method at the root $x = 0$ of the equation $x^2 e^x = 0$. Use quadratic convergent method to find second approximation x_2 to the root using $x_0 = 0.1$. Compute the absolute error.

Solution. Given $f(x) = x^2 e^x$ and so $f'(x) = (x^2 + 2x)e^x$. Using Newton's iterative formula, we get

$$x_{n+1} = x_n - \frac{(x_n^2 e^{x_n})}{(x_n^2 + 2x_n)e_n^x} = \frac{(x_n + x_n^2)}{(2 + x_n)}, \quad n \geq 0.$$

The fixed point form of the developed Newton's formula is

$$x_{n+1} = g(x_n) = \frac{(x_n + x_n^2)}{(2 + x_n)}.$$

Then

$$g(x) = \frac{(x + x^2)}{(2 + x)}, \quad g'(x) = \frac{(x^2 + 4x + 2)}{(2 + x)^2}, \quad g'(0) = \frac{1}{2} \neq 0.$$

Thus the method converges linearly to the given root.
The quadratic convergent method is modified Newton's method

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}, \qquad n \geq 0,$$

where m is the order of multiplicity of the zero of the function. To find m, we do

$$f''(x) = (x^2 + 4x + 2)e^x, \quad \text{and} \quad f''(0) = 2 \neq 0,$$

so $m = 2$. Thus

$$x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)} = x_n - 2\frac{x_n^2 e^{x_n}}{(x_n^2 + 2x_n)e_n^x} = x_n - 2\frac{x_n^2}{(x_n^2 + 2x_n)}, \qquad n \geq 0.$$

Now using initial approximation $x_0 = 0.1$, we have

$$x_1 = x_0 - 2\frac{x_0^2}{(x_0^2 + 2x_0)} = 0.00476, \quad x_2 = x_1 - 2\frac{x_1^2}{(x_1^2 + 2x_1)} = 0.0000311,$$

the required two approximations and the possible absolute error,

$$|\alpha - x_2| = |0.0 - 0.0000311| = 0.0000311.$$

Question 17: Use LU decomposition by Crout's method to find the value(s) of α for which the following matrix

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 2 & 2 \\ -2 & \alpha & 3 \end{pmatrix},$$

is singular. Compute the unique solution of the linear system $A\mathbf{x} = [2, 8, 2]^T$ by using the smallest positive integer value of α.

solution. Using the factored of the matrix A as

$$A = \mathbf{I}A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 3 \\ 4 & 2 & 2 \\ -2 & \alpha & 3 \end{pmatrix} \equiv \begin{pmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/2 & 3/2 \\ 0 & 4 & -4 \\ 0 & \alpha - 1 & 6 \end{pmatrix}$$

$$\equiv \begin{pmatrix} 2 & 0 & 0 \\ 4 & 4 & 0 \\ -2 & \alpha - 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 5 + \alpha \end{pmatrix} \equiv \begin{pmatrix} 2 & 0 & 0 \\ 4 & 4 & 0 \\ -2 & \alpha - 1 & 5 + \alpha \end{pmatrix} \begin{pmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = LU.$$

Since $|A| = |L| = 8(5 + \alpha) = 0$, gives, $\alpha = -5$, which make A singular. To find unique solution we have to choose $\alpha = 1$, and then solve the lower-triangular system

$$L\mathbf{y} = \begin{pmatrix} 2 & 0 & 0 \\ 4 & 4 & 0 \\ -2 & 0 & 6 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 2 \end{pmatrix} = \mathbf{b},$$

and it gives, $y_1 = 1, \quad y_2 = 1, \quad y_3 = 2/3$. Now solve the upper-triangular system

$$U\mathbf{x} = \begin{pmatrix} 1 & -1/2 & 3/2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2/3 \end{pmatrix} = \mathbf{y},$$

and we obtained $\mathbf{x} = [5/6, 5/3, 2/3]^T$, the unique solution of the given system.

Ps. : Mark {a, b, c or d} for the correct answer in the box. Math-254

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	b	d	b	a	c	b	a	c	c	b

Q. No.	11	12	13	14	15
a,b,c,d	b	a	c	a	b

Ps. : Mark {a, b, c or d} for the correct answer in the box. MAth-254

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	a	d	a	b	a	c	c	a	a	c

Q. No.	11	12	13	14	15
a,b,c,d	a	b	b	c	a

Ps. : Mark {a, b, c or d} for the correct answer in the box. MATh-254

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d	c	d	c	c	b	a	b	b	b	a

Q. No.	11	12	13	14	15
a,b,c,d	c	c	a	b	c