PHYS 454
2nd Midterm Exam
Saturday 8th October 2012

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Student Grade: ……./15

Please answer all questions

1. The wave function of an electron that executes an one-dimensional motion along x has, at a certain moment, the form

$$\psi = e^{3ix}$$

where x is measured in $\overset{o}{A}$.

a) What is the wavelength of this electron?

(2 marks)

b) What is the electron's velocity?

(2 marks)

Solution: A) The wavenumber is

$$k = 3\ \overset{o}{A}{}^{-1} \Rightarrow k = 3\times 10^{10}\,\text{m}^{-1} \Rightarrow \frac{2\pi}{\lambda} = 3\times 10^{10}\,\text{m}^{-1} \Rightarrow \lambda = 2.09\times 10^{-10}\,\text{m}$$

B) $$p = \frac{h}{\lambda} \Rightarrow mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{\lambda m} = \frac{6.63\times 10^{-34}}{2.09\times 10^{-10}\times 9.1\times 10^{-31}} = 0.348\times 10^{7}\,m/s$$

2. A beam of particles of energy $E = 9$ eV is directed towards a potential step (of positive potential energy) and 4% of its particles are reflected. What is the "height" V_0 of this potential?

(3 marks)

Solution: The reflection probability is

$$R = \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}}\right)^2 \Rightarrow 0.04 = \left(\frac{\sqrt{9} - \sqrt{9 - V_0}}{\sqrt{9} + \sqrt{9 - V_0}}\right)^2 \Rightarrow 0.04 = \left(\frac{3 - \sqrt{9 - V_0}}{3 + \sqrt{9 - V_0}}\right)^2$$

Put $x = \sqrt{9 - V_0}$ *and remember that x > 0.*

$$0.04 = \left(\frac{3-x}{3+x}\right)^2 \Rightarrow 0.04(3+x)^2 = (3-x)^2 \Rightarrow 0.04(x^2+6x+9) = (x^2-6x+9)$$

$$\Rightarrow 0.96x^2 - 6.24x + 8.64 = 0$$

Solving this you get

$$x_1 = 4.5, \quad x_2 = 2$$

Thus

$$4.5 = \sqrt{9 - V_0} \Rightarrow 9 - V_0 = 20.25 \Rightarrow V_0 = -11.25eV \text{ impossible}$$
$$2 = \sqrt{9 - V_0} \Rightarrow 9 - V_0 = 4 \Rightarrow V_0 = 5eV$$

3. An electron is trapped in an one-dimensional finite potential well of depth $V_0 = 11.6$ eV and of width $a = 10\overset{0}{A}$. Calculate the number of bound states.

(2 marks)

Solution: The number of bound states in a finite well is given by

$$N = \left[\frac{\lambda}{\pi/2}\right] + 1 \qquad U_0 = \frac{2mV_0}{\hbar^2} \qquad \lambda = a\sqrt{U_0}$$

$$\lambda = \frac{a\sqrt{2mV_0}}{\hbar} = \frac{10\times10^{-10}\sqrt{2\times9.1\times10^{-31}\times11.6\times1.6\times10^{-19}}}{1.055\times10^{-34}} = 17.4$$

$$N = \left[\frac{\lambda}{\pi/2}\right] + 1 = \left[\frac{17.4}{1.57}\right] + 1 = [11.08] + 1 = 12$$

4. An electron with kinetic energy *E* = 5.00 eV is incident on a barrier with thickness *L* = 0.200 nm and height V_0 = 10.0 eV. What is the probability that the electron

(a) Will tunnel through the barrier?

(2 marks)

(b) Will be reflected?

(2 marks)

Solution: The transmission probability is given by

$$T = \frac{4\left(V_0 - E\right)E}{V_0^2 \sinh^2\left(L\left(2m/\hbar^2\right)^{0.5}\sqrt{V_0 - E}\right) + 4\left(V_0 - E\right)E} =$$

$$\frac{4(10-5)5}{100\sinh^2\left(0.2\times10^{-9}\left(2\times9.1\times10^{-31}/1.11\times10^{-68}\right)^{0.5}\sqrt{5\times1.6\times10^{-19}}\right) + 4(10-5)5} =$$

$$\frac{100}{100\sinh^2\left(0.2\times10^{-9}\times1.28\times10^{19}\times8.94\times10^{-10}\right)+100} = \frac{100}{16\sinh^2\left(1.44\right)+100} =$$

$$\frac{1}{\sinh^2\left(2.288\right)+1} = \frac{1}{23.79+1} = 0.04$$

The reflection probability is

R =1-T =1-0.04=0.96

5. An electron and a proton move towards an orthogonal potential barrier with the same energy. Their energy is lower than that of the barrier. Which particle has the greater probability to go through?

(a) the electron
(b) the proton
(c) both have the same
(d) impossible to answer

(2 marks)

Correct answer is (a). The lighter a particle the easier to tunnel through.

Physical constants and formulas

$h = 6.63\times10^{-34} J\cdot s$, $\hbar = h/2\pi = 1.055\times10^{-34} J\cdot s$, $1\overset{o}{A} = 10^{-10} m$, $m_e = 9.1\times10^{-31}$ kg, $1eV = 1.6\times10^{-19}$ J

For a particle wave: $\lambda = h/p$ and $k = 2\pi/\lambda$.

For an finite square well:

$$N = \left[\frac{\lambda}{\pi/2}\right] + 1 \qquad U_0 = \frac{2mV_0}{\hbar^2} \qquad \lambda = a\sqrt{U_0}$$

For a potential step: $E > V_0$

$$R = \left(\frac{k - k^{'}}{k + k^{'}} \right)^2 = \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}} \right)^2, \quad T = \frac{4kk^{'}}{\left(k + k^{'}\right)^2} = \frac{4\sqrt{E\left(E - V_0\right)}}{\left(\sqrt{E} + \sqrt{E - V_0}\right)^2}$$

For a potential barrier of finite width: $E > V_0$

$$T = \frac{4E\left(E - V_0\right)}{V_0^2 \sin^2\left(L\left(2m/\hbar^2\right)^{0.5}\sqrt{E - V_0}\right) + 4E\left(E - V_0\right)}$$

For a potential barrier of finite width: $E < V_0$

$$T = \frac{4\left(V_0 - E\right)E}{V_0^2 \sinh^2\left(L\left(2m/\hbar^2\right)^{0.5}\sqrt{V_0 - E}\right) + 4\left(V_0 - E\right)E}$$

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$