4.Random Variables (Pages 8-16)

Q1: (a) S ={HHH ,HHT,HTH,THH,TTH,HTT,THT,TTT}

(b)

Values of x = -3,-1,1,3

S	X=number of heads- number of tails
HHH	X =3 - 0 = 3
HHT	X = 2 - 1 = 1
HTH	X = 2 - 1 = 1
THH	X = 2 - 1 = 1
TTH	X = 1 - 2 = -1
HTT	X = 1 - 2 = -1
THT	X = 1 - 2 = -1
TTT	X = 0 - 3 = -3

(c)

X	-3	-1	1	3	total
.f(x)	1/8	3/8	3/8	1/8	1

(d)$P(X\le1)$ = 1/8+3/8+3/8 =7/8

(e)$P(X<1)$ = 1/8+3/8 =4/8=1/2

(f)

X	-3	-1	1	3	total
.f(x)	1/8	3/8	3/8	1/8	1
Xf(x)	-3/8	-3/8	3/8	3/8	0 =E(x)
X^2 f(x)	9/8	3/8	3/8	6/8	21/8=E(X^2)

(F) mean =E(x) = 0

(g) $V(x) = E(X^2) – [E(X)]^2 = 21/8 - 0^2 = 21/8$

Q2:

X	0	1	2	total
.f(x)	0.64	0.32	0.04	1

4.P(at least one) = $P(X \ge 1)$ = 0.32+0.04 =0.36

5. P(at most one) = $P(X \le 1)$ = 0.64+0.32 =0.96

6.

X	0	1	2	total
.f(x)	0.64	0.32	0.04	1
Xf(x)	0	0.32	0.08	0.40=E(x)
X^2 f(x)	0	0.32	0.16	0.48=E(X^2)

6. Mean = E(X) = 0.40

7. V(x) = $E(X^2) - [E(X)]^2 = 0.48 - 0.40^2 = 0.32$

8.Standard deviation = $\sqrt{V(x)} = \sqrt{0.32} = 0.5657$

Q4.

X	-3	6	9	total
.f(x)	0.1	0.5	0.4	1
Xf(x)	-0.3	3	3.6	6.3=E(x)
X^2 f(x)	0.9	18	32.4	51.3=$E(X^2)$

1. Mean = E(X) = 6.3
2. $E(X^2) = 51.3$
3. V(x) = $E(X^2) - [E(X)]^2 = 51.3 - 6.3^2 = 11.61$

   Standard deviation = $\sqrt{V(x)} = \sqrt{11.61} = 3.41$
4. E(2X +1) = 2E(X) + 1 = 2x(6.3) + 1 =13.6
5. V(2X +1) = 2^2V(X) + V(1) = 4x 11.61 +0 = 46.44

Q5 : (A)

X	0	1	2	3	4	total
.f(x)	1/10	2/10	3/10	4/10	5/10	1

(B)

X	0	1	2	3	4	total
.f(x)	-1/5	0	1/5	2/5	3/5	1

(C)

X	0	1	2	3	4	total
.f(x)	1/5	1/5	1/5	1/5	1/5	1

(D)

X	0	1	2	3	total
.f(x)	5/6	4/6	1/6	-1/6	1

Q7: (1)

X	1	2	3	total
.f(x)	k	2k	3k	1

(2) F(x) =P(X ≤ x) = $\begin{cases} 0 & x < 1 \\ \frac{1}{6} & 1 \le x < 2 \\ \frac{3}{6} & 2 \le x < 3 \\ \frac{6}{6} & 3 \le x \end{cases}$

(3) P(0.5 < X ≤ 2.5) =1/6 + 2/6 = 3/6

Q9:

X	0	1	2	3	total
.f(x)	0.4	C	0.3	0.1	1

0.4+ c+ 0.3 +0.1 =1

0.8+ c = 1   Then , c = 0.2

Q12:   F(x) =P(X ≤ x) = $\begin{cases} 0 & x < 0 \\ \frac{1}{16} & 0 \le x < 1 \\ \frac{5}{16} & 1 \le x < 2 \\ \frac{11}{16} & 2 \le x < 3 \\ \frac{15}{16} & 3 \le x < 4 \\ 1 & 4 \le x \end{cases}$

(1) Find f(x) :

X						total
.f(x)						1

(2) P(X=2) = 6/16 =3/8

(3) P(2 ≤ X ≤ 4) = F(4) – F(1) = 1 – 5/16 = 11/16

Q13:

Mean of X = E(x) = 10

V(x) =4

(a) Mean of Y : E(Y) = E(2X -2) = E(2X) – 2 = 2x10 -2 = 18

(b) Variance of Y : V(Y) = V(2X -2) = V(2X) – V(2) = 2^2x4 -0 = 4x4 = 16

Q15: f(-1) =0.05 , f(0) =0.25 , f(1) = 0.25 , f(2) = 0.45

X	-1	0	1	2	total
.f(x)	0.05	0.25	0.25	0.45	1

(1) P(X <1) = 0.05+ 0.25 = 0.30 (A)
(2) P(X ≤1) = 0.05+ 0.25 + 0.25 = 0.55 (B)
(3) E(X) = 1.1 (A)
(4) E(X^2) = 2.10 (B)
(5) Var(X) = 0.89
(6) F(1) = P(X ≤1) = 0.05+ 0.25 + 0.25 = 0.55 (D)

4.2 Continouse Distribution

Q1: μ = 16 , σ^2 = 5 , then P(X =16) = 0 (C)

Q2: f(x) $k\sqrt{x}$, $0 < x < 1$

1) $\int_0^1 k\sqrt{x}\,dx = 1$ ⟶ $\int_0^1 k\sqrt{x}\,dx = \quad k\left.\frac{x^{3/2}}{3/2}\right]_0^1 = 1$

$\frac{2}{3}\,k\,[(1-0)] = 1$ ⟶ k = 3/2 =1.5

2) P(0.3 < x <0.6) = $\int_{0.3}^{0.6}\frac{3}{2}\sqrt{x}\,dx = \frac{3}{2}\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_{0.3}^{0.6} = 0.6^{\frac{3}{2}} - 0.3^{\frac{3}{2}} = 0.3004$

3) $E(x) = \int_0^1 \frac{3}{2}\,x\sqrt{x}\,dx = \int_0^1 \frac{3}{2}x^{\frac{3}{2}}\,dx = \int_0^1 \frac{3}{2}x^{\frac{5}{2}}\,dx = \frac{3}{2}\left.\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1 \quad = \frac{3}{2}x\frac{2}{7} = \frac{3}{7}$

Q3:

f(x) $k(x+1)$, $0 < x < 2$

1) $\int_0^2 k(x+1)\,dx = 1$ ⟶ $\int_0^2 k(x+1)\,dx = k\left[\frac{x^2}{2} + x\right]_0^2 = 1$

$k\left[\left(\frac{4}{2} + 2\right)\right] = 1$ ⟶ k(4) = 1 , Then k = 1/4

2) P(0<X ≤ 1) = $\int_0^1 \frac{1}{4}(x+1)\,dx = \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^1 = \frac{1}{4}\left(\frac{1}{2} + 1\right) = \frac{3}{8} = 0.375$

3) F(x) = P(X ≤ x) = $\int_0^x \frac{1}{4}(x+1)\,dx = \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^x = \frac{1}{4}\left(\frac{x^2}{2} + x\right) = \frac{x^2 + 2x}{8}$

4) P(0 < X ≤ 1) = F(1) – F(0) = $\frac{1^2 + 2(1)}{8} - \frac{0}{8} = \frac{3}{8}$

Q9 :

(1) P(0 < X < 2) = F(2) – F(0) = $\frac{2}{2+1} = \frac{2}{3} - 0 = \frac{2}{3} = 0.667\ (c)$

(2) P(X ≤ k) = F(k) = 0.5

$\frac{k}{k+1} = 0.5 \quad Then\ k = 0.5k + 0.5 \quad , \quad k = 1$

4.3 CHYBYSHEV'S THEORM

Q2: $\mu = 12$, $\sigma^2 = 9$, $\sigma = 3$

$P(3 < X < 21)$

$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

To find value of k : $\mu + k\sigma = 21$

$12 + 3k = 21$ Then $3k = 21 - 12 = 9$, So $k = 9/3 = 3$

Then lower bound is

$$1 - \frac{1}{k^2} = 1 - \frac{1}{3^2} = \frac{8}{9}$$

Q4: $\mu = 5$, $\sigma = 2.89$, $f(x) = 1/10$ $0<x<10$

1. $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = P(5 - 1.5(2.89) < X < 5 + 1.5(2.89)) =$

   $P(5 - 4.335 < X < 5 + 4.335) = P(0.665 < x < 9.335) = \int_{0.665}^{9.335} \frac{1}{10}\, dx =$
   $\frac{1}{10}(9.335 - 0.665) = 0.867$

2. $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = P(0.665 < x < 9.335)$

   $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

To find value of k : $\mu + k\sigma = 9.335$

$5 + 2.8k = 9.335$ Then $2.89k = 9.335 - 5$, So $k = 4.335/3 = 1.45$

Then lower bound is

$$1 - \frac{1}{k^2} = 1 - \frac{1}{1.45^2} = 0.524$$

Q1:

3.2: From book :

B:Blemished , N Non-Blemished

(a) S ={BBB ,BBN,BNB,NBB,NNB,NBN,BNN,NNN}

Values of x = 0 ,1 ,2 ,3

S	X=number of Blemished
BBB	X = 3
BBN	X =2
BNB	X =2
NBB	X =2
NNB	X =1
NBN	X =1
BNN	X =1
NNN	X =0

3.14:

_ F(x)= $1 - e^{-8x}$ $x \geq 0$

P(less than 12) = P(X ≤ 12) = F(12) = $1 - e^{-8(12)} = 1$

3.12:

Average = mean =

$$E(x) = \int_0^1 2\,x(1-x)\,dx = 2\int_0^1 x - x^2\,dx = 2[\frac{x^2}{2} - \frac{x^3}{3}\Big]_0^1 = \frac{1}{3}$$
