(8 One-and Two-Sample Test Of Hypothesis)

Single Mean:

Q1) Suppose that we are interested in making some statistical inferences about the mean, μ, of a normal population with standard deviation $\sigma = 2.0$. Suppose that a random sample of size *n*=49 from this population gave a sample mean $\bar{X} = 4.5$.

(1) If we want to test $H_0\colon \mu = 5.0$ against $H_1\colon \mu \neq 5.0$, then the test statistic equals to

(A) Z=-1.75 (B) Z=1.75 (C) T=-1.70 (D) T=1.70 (E) Z=-1.65

Ans.: $Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{4.5-5}{2/\sqrt{49}} = -1.75.$

(2) If we want to test $H_0\colon \mu = 5.0$ against $H_1\colon \mu > 5.0$ at $\alpha = 0.05$, then the Rejection Region of $H_0$is

(A) $(1.96, \infty)$ (B) $(2.325, \infty)$ (C) $(-\infty, 1.645)$(D) $(-\infty, 1.96)$ (E) $(1.645, \infty)$

Ans.: Rejection Region$(Z > Z_{1-\alpha}) = (Z > Z_{0.95}) = (1.645, \infty)$

(3) If we want to test $H_0\colon \mu = 5.0$ against $H_1\colon \mu > 5.0$ at $\alpha = 0.05$, then we

(A) Accept H_0 (B) Reject H_0

Ans.: Since -1.75$\notin$ $(1.645, \infty)$ Accept H_0

Or we can calculate P-value=P(Z>-1.75)=1-P(Z<-1.75)=1-0.0401=0.9599>0.05

Q2 HW

Q3) An electrical firm manufactures light bulbs that have a length of life that is normally distributed. A sample of 20 bulbs were selected randomly and found to have an average of 655 hours and a sample standard deviation of 27 hours. Let μ be the population mean of life lengths of all bulbs manufactured by this firm. Test $H_0\colon \mu = 660$against $H_1\colon \mu \neq 660$? Use a 0.02 level of significance.

Ans.:

$n = 20, \bar{X} = 655, S = 27$

$H_0\colon \mu = 660$ against $H_1\colon \mu \neq 660$

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{655 - 660}{27/\sqrt{20}} = -0.828$$

Rejection Region $\left(T > T_{n-1,1-\alpha/2}\right) or \left(T < -T_{n-1,1-\alpha/2}\right)$

$$\alpha = 0.02, \quad \frac{\alpha}{2} = 0.01$$

$T_{n-1,1-\alpha/2} = T_{19,0.99} = 2.539$

Rejection Region$(T > 2.539)$ or $(T < -2.539) \Longrightarrow Not\ reject\ H_0$.

Q4+Q5+Q6+Q7 HW

Q 10.20 Page 356

A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a sample standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

Ans.:

$n = 64, \bar{X} = 5.23,\ \ S = 0.24, \alpha = 0.05$

$H_0: \mu\ = 5.5$ against $H_1: \mu < 5.5$

$$Z = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{5.23 - 5.5}{0.24/\sqrt{64}} = -9$$

Rejection Region $(Z < -Z_{1-\alpha})$

$\alpha = 0.05, \qquad 1 - \alpha = 0.95$

$Z_{1-\alpha} = Z_{0.95} = 1.645$

Reject H_0 $if\ Z < -Z_{1-\alpha} \Longrightarrow Since - 9 < -1.645\ we\ Reject\ H_0$.

Or we can use $p - value = P(Z < -9) = 0.00 < \alpha \Longrightarrow Reject\ H_0$.

Q 10.23 Page 356

Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Ans.:

$n = 10, \bar{X} = 10.06, \qquad s = 0.245, \qquad \alpha = 0.01$

$H_0: \mu\ = 10$ against $H_1: \mu \neq 10$

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{10.06 - 10}{0.245/\sqrt{10}} = 0.775$$

Rejection Region$\left(T > T_{n-1,1-\alpha/2}\right)$ or $\left(T < -T_{n-1,1-\alpha/2}\right)$

$\alpha = 0.01, \quad \frac{\alpha}{2} = 0.005$

$T_{n-1,1-\alpha/2} = T_{9,0.995} = 3.25$

Reject H_0 $if\ T < -3.25\ or\ T > 3.25 \Longrightarrow Not\ Reject\ H_0.$

H.W: 10.24, 10.25

<u>Two Means:</u>

Q1) Two random samples were independently selected from two normal populations with equal variances. The results are summarized as follows.

	First Sample	Second Sample
sample size (n)	12	14
sample mean ($\bar{X}$)	10.5	10.0
sample variance (S^2)	4	5

Let $\mu_1$and $\mu_1$be the true means of the first and second populations, respectively. Test $H_0: \mu_1 = \mu_2$ against $H_0: \mu_1 \neq \mu_2$. (use $\alpha = 0.05$)

Ans:.

The test statistic

$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Pooled variance = $S_P^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{(12-1)(4)+(14-1)(5)}{12+14-2} = 4.542$

$S_p = \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}} = \sqrt{4.542}$ =2.131

$H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(10.5 - 10.0) - 0}{(2.131)\sqrt{\frac{1}{12} + \frac{1}{14}}} = 0.5964$$

Rejection Region $\left(T < -T_{n_1+n_2-2,1-\alpha/2}\right)\ or\ \left(T > T_{n_1+n_2-2,1-\alpha/2}\right)$

$T_{n_1+n_2-2,1-\frac{\alpha}{2}} = T_{24,0.975} = 2.064$

Reject H_0 $if\ T < -2{,}064\ or\ T > 2.064 \Longrightarrow Snot\ Reject\ H_0.$

Q2+Q3+Q4 HW

Q5) To determine whether car ownership affects a student's academic achievement, two independent random samples of 100 male students were each drawn from the students' body. The first sample is for non-owners of cars and the second sample is for owners of cars. The grade point average for the 100 non-owners of cars had an average equals to 2.70, while the grade point average for the 100 owners of cars had an average equals to 2.54. Do data present sufficient evidence to indicate a difference in the mean achievement between car owners and non-owners of cars? Test using $\alpha = 0.05$. Assume that the two populations have variances $\sigma^2_{non-owner} = 0.36$and $\sigma^2_{owner} = 0.40$.

Ans:.
$n_1 = 100, \quad \bar{X}_1 = 2.7, \quad \sigma_1^2 = 0.36$
$n_2 = 100, \quad \bar{X}_2 = 2.54, \quad \sigma_2^2 = 0.4$
$\alpha = 0.05$

$H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(2.7 - 2.54) - 0}{\sqrt{\frac{0.36}{100} + \frac{0.4}{100}}} = 1.835$$

Reject H_0 if $(Z < -Z_{1-\alpha/2})$ or $(Z > Z_{1-\alpha/2})$

$\alpha = 0.05, \quad \frac{\alpha}{2} = 0.025, \; 1 - \frac{\alpha}{2} = 0.975$

$Z_{1-\alpha/2} = Z_{0.975} = 1.96$

Reject H_0 $if\ Z < -1.96\ or\ Z > 1.96 \Longrightarrow We\ cannot\ Reject\ H_0.$

Q 10.30 Page 357
Random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation$\sigma_2 = 3.4$, has a mean $\bar{x}_2 = 76$. Test the hypothesis that $\mu_1 = \mu_2$ against the alternative, $\mu_1 \neq \mu_2$. “Use a significance level of 0.05.”

Solution:
$n_1 = 25\,, \bar{x}_1 = 81, \sigma_1 = 5.2$
$n_2 = 36\,, \bar{x}_2 = 76, \sigma_1 = 3.4$
$H_0: \mu_1 = \mu_2$against$H_1: \mu_1 \neq \mu_2$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - d}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(81 - 76) - 0}{\sqrt{\frac{(5.2)^2}{25} + \frac{(3.4)^2}{36}}} = 4.222$$

Reject H_0 if$(Z < -Z_{1-\alpha/2})$ or $(Z > Z_{1-\alpha/2})$

$\alpha = 0.05, \quad \frac{\alpha}{2} = 0.025, \; 1 - \frac{\alpha}{2} = 0.975$

$Z_{1-\alpha/2} = Z_{0.975} = 1.96$

Reject H_0 $if\ Z < -1.96\ or\ Z > 1.96 \Longrightarrow Reject\ H_0.$

$p - value = 2P(Z > 4.22) = 2[1 - P(Z < 4.22)] = 2(1 - 1) = 0.00$

Q 10.33 Page 357

A study was conducted to see if increasing the substrate concentration has an appreciable effect on the velocity of a chemical reaction. With a substrate concentration of 1.5 moles per liter, the reaction was run 15 times, with an average velocity of 7.5 micromoles per 30 minutes and a standard deviation of 1.5. With a substrate concentration of 2.0 moles per liter, 12 runs were made, yielding an average velocity of 8.8 micromoles per 30 minutes and a sample standard deviation of 1.2. Is there any reason to believe that this increase in substrate concentration causes an increase in the mean velocity of the reaction of more than 0.5 micromole per 30 minutes? Use a 0.01 level of significance and assume the populations to be approximately normally distributed with equal variances.

Solution:

$n_1 = 15, \bar{X}_1 = 7.5, \; S_1 = 1.5$

$n_2 = 12, \bar{X}_1 = 8.8, \; S_1 = 1.2, \qquad \alpha = 0.01$

Pooled variance = $S_P^2 = \frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2} = \frac{(1.5)^2(15-1)+(1.2)^2(12-1)}{15+12-2} = 1.8936$

$$S_P = \sqrt{\frac{S_1^2(n_1-1)+S_2^2(n_2-1)}{n_1+n_2-2}} = \sqrt{\frac{(1.5)^2(15-1)+(1.2)^2(12-1)}{15+12-2}} = 1.3761$$

$H_0: \mu_1 - \mu_2 = 0.5 \text{against} H_1: \mu_1 - \mu_2 > 0.5$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - d}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(7.5 - 8.8) - 0.5}{\sqrt{1.8936}\sqrt{\frac{1}{15} + \frac{1}{12}}} = -3.377$$

Rejection Region$\left(T > T_{n_1+n_2-2,1-\alpha}\right)$

$T_{n_1+n_2-2,1-\alpha} = T_{25,0.99} = 2.485$

Reject H_0 $if\ T > 2.485 \; \Longrightarrow Cannot\ Reject\ H_0.$

H.W: 10.29, 10.35

Single Proportion:

Q1) A researcher was interested in making some statistical inferences about the proportion of smokers (p) among the students of a certain university. A random sample of 500 students showed that 150 students smoke.

Ans:.

$n = 500, X = 150.$

(1) If we want to test $H_0: p = 0.25$ against $H_1: p \neq 0.25$ then the test statistic equals to

(A) Z=2.2398 (B) T=-2.2398 (C) Z=-2.4398 (D) Z=2.582 (E) T=2.2398

$$p = \frac{X}{n} = \frac{150}{500} = 0.3.$$

$$Z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.3 - 0.25}{\sqrt{\frac{0.25*0.75}{500}}} = 2.582$$

(2) If we want to test $H_0: p = 0.25$ against $H_1: p \neq 0.25$ at $\alpha = 0.1$, then the Acceptance Region of H_0 is

(A) $(-1.645, \infty)$ (B) $(-\infty, 1.645)$ (C) $(-1.645, 1.645)$ (D) $(-1.285, \infty)$ (E) $(-1.285, 1.285)$

Ans:. The acceptance region is $-Z_{1-\alpha/2} < Z < Z_{1-\alpha/2}$

$\alpha = 0.1 \rightarrow Z_{1-\alpha/2} = Z_{0.95} = 1.645.$

Thus, the acceptance region is -1.645<Z<1.645.

(3) If we want to test $H_0: p = 0.25$ against $H_1: p \neq 0.25$ at $\alpha = 0.1$, then we

(A) Accept H_0 (B) Reject H_0

Since $Z = 2.582 > 1.645$.

Q10.58 Page 365

It is believed that at least 60% of the residents in a certain area favor an annexation suit by a neighboring city. What conclusion would you draw if only 110 in a sample of 200 voters favored the suit? Use a 0.05 level of significance.

Solution:

$n = 200 \quad , \alpha = 0.05$

Thus,

$p = \frac{110}{200} = 0.55.$

$H_0: p = 0.6$ against $H_1: p > 0.6$

$$Z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.55 - 0.6}{\sqrt{\frac{0.6*0.4}{200}}} = -1.443$$

Reject H_0 if $(Z > Z_{1-\alpha})$

$Z_{1-\alpha} = Z_{0.95} = 1.645$

Reject H_0 $if\ Z > 1.645 \implies We\ Cannot\ Reject\ H_0$.

$p - value = P(Z > -1.443) = 1 - 0.0749 = 0.9251 > \alpha \implies Not\ Reject\ H_0$

Q10.59 Page 365

A fuel oil company claims that one-fifth of the homes in a certain city are heated by oil. Do we have reason to believe that fewer than one-fifth are heated by oil if, in a random sample of 1000 homes in this city, 136 are heated by oil? Use a *P*-value in your conclusion.

$n = 1000, X = 136.$

Thus,

$p = \frac{136}{1000} = 0.136.$

$H_0: p = 0.2$ against $H_1: p < 0.2$

$$Z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.136 - 0.2}{\sqrt{\frac{0.2*0.8}{1000}}} = -5.06$$

$p - value = P(Z < -5.06) = 0.00 \implies Reject\ H_0$