Some Discrete Probability Distributions

DISCRETE UNIFORM DISTRIBUTION

Q1. Let the random variable X have a discrete uniform with parameter *k*=3 and with values 0,1, and 2. Then:

$$f(x) = \frac{1}{3}\ ; x = 0, 1, 2$$

(a) P(X=1) is

(A) 1.0 (B) 1/3 (C) 0.3 (D) 0.1. (E) None

(b) The mean of X is:

(A) 1.0 (B) 2.0. (C) 1.5 (D) 0.0 (E) None

$$\mu = \frac{\sum x_i}{k} = \frac{0+1+2}{3} = 1$$

(c) The variance of X is: $\sigma^2 = \frac{\sum(x_i-\mu)^2}{k} = \frac{(0-1)^2+(1-1)^2+(2-1)^2}{3} = \frac{2}{3}$

(A) 0/3=0.0 (B) 3/3=1.0 (C) 2/3=0.67 (D) 4/3=1.33 (E) None

BINOMIAL DISTRIBUTION

Q3. Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen.

$p = 0.4\,,\ n = 10. \quad , f(x) = \binom{10}{x}(0.4)^x(0.6)^{10-x}; x = 0, 1, 2, 3, \ldots, 10$

(1) What is the expected number of persons who will die in this sample?

$E(X) = np = 10\,(0.4) = 4$

(2) What is the variance of the number of persons who will die in this sample?

$V(X) = npq = 10\,(0.4)(0.6) = 2.4$

(3) What is the probability that exactly 4 persons will die among this sample?

$P(X = 4) = f(4) = \binom{10}{4}(0.4)^4(0.6)^6 = 0.2142$

(4) What is the probability that less than 3 persons will die among this sample?

$$\boldsymbol{P(X < 3) = f(2) + f(1) + f(0) = 0.1673}$$

(5) What is the probability that more than 8 persons will die among this sample?

$$\boldsymbol{P(X > 8) = p(X \geq 9) = f(9) + f(10) = 0.0017}$$

Q7. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

X=number of a green balls.

(With replacement) $f(x) = \binom{3}{x}\left(\frac{2}{6}\right)^{x}\left(\frac{4}{6}\right)^{3-x}; x = 0, 1, 2, 3$

$$f(2) = \binom{3}{2}\left(\frac{2}{6}\right)^{2}\left(\frac{4}{6}\right)^{1}$$

(A) 6/27. **(B) 2/27.** **(C) 12/27.** **(D) 4/27**

Q9. If X~Binomial(n,p), E(X)=1, and Var(X)=0.75, find P(X=1).

$$E(X) = np = 1$$

$$Var(X) = npq = 0.75$$

$$q = 0.75 \Longrightarrow p = 0.25 \Longrightarrow n = 4$$

$$f(x) = \binom{4}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{3} = 0.4219$$

Q11. A traffic control engineer reports that 75% of the cars passing through a checkpoint are from Riyadh city. If at this checkpoint, five cars are selected at random.

$$p = 0.75\,,\ n = 5.\quad , f(x) = \binom{5}{x}(0.75)^{x}(0.25)^{5-x}; x = 0, 1, 2, 3, 4, 5$$

(1) The probability that none of them is from Riyadh city equals to:

$$f(0) = p(X = 0) = \binom{5}{0}(0.75)^{0}(0.25)^{5}$$

(A) 0.00098 (B) 0.9990 (C) 0.2373 (D) 0.7627

(2) The probability that four of them are from Riyadh city equals to:

$$f(0) = p(X = 4) = \binom{5}{4}(0.75)^4(0.25)^1 = 0.3955$$

(A) 0.3955 (B) 0.6045 (C) 0 (D) 0.1249

(3) The probability that at least four of them are from Riyadh city equals to:

$$p(X \geq 4) = f(4) + f(5) = 0.6328$$

(A) 0.3627 (B) 0.6328 (C) 0.3955 (D) 0.2763

(4) The expected number of cars that are from Riyadh city equals to:

$E(X) = np = 5\,(0.75) = 3.75$

(A) 1 (B) 3.75 (C) 3 (D) 0

H.W: Q1, Q2,Q4, Q5, Q6, Q8,
Q10 Deleted

HYPERGEOMETRIC DISTRIBUTION

Q1. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets.

(i) Find the probability distribution function of the random variable X representing the number of defective sets purchased by the hotel.

X: number of defective sets purchased by the hotel. , k= 2 ,N=7, n=3

$$f(x) = \frac{(2C_x)\,(5C_{3-x})}{7C_3}$$

(ii) Find the probability that the hotel purchased no defective television sets.

$$f(0) = \frac{(2C_0)\,(5C_{3-0})}{7C_3} = \frac{2}{7} = 0.2857$$

(iii) What is the expected number of defective television sets purchased by the hotel?

$$E(X) = \frac{n*k}{N} = \frac{3*2}{7} = 0.8571$$

(iv) Find the variance of X. $V(X) = \frac{n*k(N-k)(N-n)}{N^2(N-1)} = \frac{3*2(7-2)(7-3)}{7^2(7-1)} = \frac{20}{49} = 0.4082$

Q4. A box contains 2 red balls and 4 black balls. Suppose that a sample of 3 balls were selected randomly and without replacement. Find,

1. **The probability that there will be 2 red balls in the sample.**

X: number of red palls in sample . , k= 2 ,N=6, n=3

$$f(x) = \frac{(2C_x)(4C_{3-x})}{6C_3}$$

$$f(2) = \frac{(2C_2)(4C_{3-2})}{6C_3} = \frac{1}{5} = 0.2$$

2. The probability that there will be 3 red balls in the sample.

$p(X = 3) = 0$, because $0 \le x \le k$

3. The expected number of the red balls in the sample.

$$E(X) = \frac{n*k}{N} = \frac{3*2}{6} = 1$$

Q10. A box contains 4 red balls and 6 green balls. The experiment is to select 3 balls at random. Find the probability that all balls are red for the following cases:

X: number of red palls.

(1) If selection is without replacement

k= 4 ,N=10 , n=3 , $f(x) = \frac{(4C_x)(6C_{3-x})}{10C_3}$

$\text{since } n = 3 \text{ then all red balls in sample is } 3$

$f(3) = \frac{(4C_3)(6C_{3-3})}{10C_3} = \frac{1}{30} = 0.0333$

(A) 0.216 (B) 0.1667 (C) 0.6671 (D) 0.0333

(2) If selection is with replacement

$p = \frac{4}{10} = 0.4$, $q = 0.6, n = 3$, $f(x) = \binom{3}{x}(0.4)^x(0.6)^{3-x}, X = 0, 1, 2, 3$

$$f(3) = \binom{3}{3}(0.4)^3(0.6)^0 = 0.064$$

(A)0.4600 (B) 0.2000 (C) 0.4000 (D) 0.0640

H.W:Q2 ,Q3, Q5,Q9 ;

Deleted Q6,Q7 ,Q8

POISSON DISTRIBUTION

Q2. At a checkout counter, customers arrive at an average of 1.5 per minute. Assuming Poisson distribution, then

$$f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}; x = 0, 1, 2, \ldots$$

(1) The probability of no arrival in two minutes is

$$2\lambda = 2 * 1.5 = 3 \quad then \quad f(0) = \frac{(3)^0 e^{-3}}{0!} = 0.4978$$

(A)0.0 (B) 0.2231 (C) 0.4463 (D) 0.0498 (E) 0.2498

(2) The variance of the number of arrivals in two minutes is

$$Since\ V(x) = \lambda t = 1.5 \quad then \quad V(x) = 2\lambda = 2 * 1.5 = 3$$

(A) 1.5 (B) 2.25 (C) 3.0 (D) 9.0 (E) 4.5

Q3. Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 4 calls per day.

$$\mu = \lambda t = 4$$

(a) The probability that 2 calls will be received in a given day is

$$f(2) = \frac{(4)^2 e^{-4}}{2!} = 0.146525$$

(A)0.546525 (B) 0.646525 (C) 0.146525 (D) 0.746525

(b) The expected number of telephone calls received in a given week is

(c)$\mu = \lambda t = 4 * 7 = 28$

(A) 4 (B) 7 (C) 28 (D) 14

(c) The probability that at least 2 calls will be received in a period of 12 hours is

To convert 12 hours to day >> 12/24=1/2 =0.5 , Then $\lambda t = 4 * 0.5 = 2$

$$p(x \geq 2) = 1 - p(x < 2) = 1 - p(x \leq 1) = 1 - \sum_{x=0}^{1} \frac{(2)^x e^{-2}}{x!} = \frac{(2)^0 e^{-2}}{0!} + \frac{(2)^1 e^{-2}}{1!} = 0.59399$$

(A) 0.59399 (B) 0.19399 (C) 0.09399 (D) 0.29399

H.W : Q1 ,Q4, Q 5, Q7 ,Q8
Delete Q6

Table A.2 Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6							1.0000	1.0000	1.0000

	μ								
r	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8		1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9			1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10				0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11				1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12					1.0000	0.9999	0.9997	0.9992	0.9980
13						1.0000	0.9999	0.9997	0.9993
14							1.0000	0.9999	0.9998
15								1.0000	0.9999
16									1.0000

H.W

Q4. Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population. $p = 0.5\ ,\ \ n = 3$

$$f(x) = \binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x} \quad ; x = 0,1,2,3$$

(a) The probability that no females are selected is $f(0) = \binom{3}{0}\left(\frac{1}{2}\right)^{3}$

(A) 0.000 (B) 0.500 (C) 0.375 (D) 0.125

(b) The probability that at most two females are selected is $P(X \leq 2) = 1 - f(3) = 1 - 0.125$

(A) 0.000 (B) 0.500 (C) 0.875 (D) 0.125

(c) The expected number of females in the sample is $E(X) = np = \frac{3}{2} = 1.5$

(A) 3.0 (B) 1.5 (C) 0.0 (D) 0.50

(d) The variance of the number of females in the sample is $\sigma^2 = Var(X) = npq = 3\ \frac{1}{2}\frac{1}{2} = 0.75$

(A) 3.75 (B) 2.75 (C) 1.75 (D) 0.75

Q1. Suppose that 4 out of 12 buildings in a certain city violate the building code (تنتهك حقوق البناء). A building engineer randomly inspects a sample of 3 new buildings in the city.

$$p = \frac{4}{12} = \frac{1}{3}, \qquad n = 3$$

(a) Find the probability distribution function of the random variable X representing the number of buildings that violate the building code in the sample.

$$f(x) = \binom{3}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{3-x} \quad ; x = 0,1,2,3$$

(b) Find the probability that:

(i) none of the buildings in the sample violating the building code. $f(0) = \binom{3}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{3} = 0.2963$

(ii) one building in the sample violating the building code. $f(1) = \binom{3}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{2} = 0.4444$

(iii) at lease one building in the sample violating the building code. $P(X \geq 1) = 1 - f(0) = 1 - 0.2963 = 0.7037$

(c) Find the expected number of buildings in the sample that violate the building code (E(X)).

$$E(X) = np = \frac{3}{3} = 1$$

(d) Find $\sigma^2 = Var(X)$. $\sigma^2 = Var(X) = npq = 3\,\frac{1}{3}\frac{2}{3} = \frac{2}{3} = 0.6667$

Q2. Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

X=number of girls., k=3, N=5, n=2

$$f(x) = \frac{{}_3C_x\,{}_2C_{2-x}}{{}_5C_2}; x = 0,1,2$$

(a) The probability that no girls are selected is $f(0) = \frac{{}_3C_0\,{}_2C_2}{{}_5C_2}$

(A) 0.0 (B) 0.3 (C) 0.6 (D) 0.1

(b) The probability that at most one girls are selected is $P(X \leq 1) = f(0) + f(1)$

$= \frac{{}_3C_0\,{}_2C_2}{{}_5C_2} + \frac{{}_3C_1\,{}_2C_1}{{}_5C_2}$

(A) 0.7 (B) 0.3 (C) 0.6 (D) 0.1

(c) The expected number of girls in the sample is $E(X) = \frac{n*k}{N} = \frac{2*3}{5}$

(A) 2.2 (B) 1.2 (C) 0.2 (D) 3.2

(d) The variance of the number of girls in the sample is

$$V(X) = \frac{n * k(N-k)(N-n)}{N^2(N-1)} = \frac{2 * 3(5-3)(5-2)}{5^2(5-1)}$$

(A) 36.0 (B) 3.6 (C) 0.36 (D) 0.63

Q5. From a lot of 8 missiles, 3 are selected at random and fired. The lot contains 2 defective missiles that will not fire. Let X be a random variable giving the number of defective missiles selected.

X= number of defective missiles, k=2, N=8, n=3

1. Find the probability distribution function of X.

$f(x) = \frac{{}_2C_x\,{}_6C_{3-x}}{{}_8C_3}; x = 0,1,2 = \min(k, n)$

2. What is the probability that at most one missile will not fire?

$P(X \leq 1) = f(0) + f(1) = \frac{{}_2C_0\,{}_6C_3}{{}_8C_3}+\frac{{}_2C_1\,{}_6C_2}{{}_8C_3}$=0.8928

3. Find E(X) and Var(X).

$E(X) = \frac{n*k}{N} = \frac{3*2}{8}$=0.75

$V(X) = n\frac{k}{N}\left(1-\frac{k}{N}\right)\frac{N-n}{N-1} = 3\frac{2}{8}\left(1-\frac{2}{8}\right)\frac{8-3}{8-1} = 0.4018$

H.W: Q4

Q8. The number of faults in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

$$f(x) = \frac{(0.6t)^x}{x!} e^{-0.6t}; x = 0,1,2, \dots$$

(1) The probability of 2 faults per 100 feet of such cable is: $f(2) = \frac{(0.6)^2}{2!} e^{-0.6}$

(A) 0.0988 (B) 0.9012 (C) 0.3210 (D) 0.5

(2) The probability of less than 2 faults per 100 feet of such cable is:

$P(X < 2) = P(X \leq 1)$=$\sum_{x=0}^{1} p(x, 0.6) = \frac{(0.6)^1}{1!} e^{-0.6}$+$\frac{(0.6)^0}{0!} e^{-0.6}$

(A) 0.2351 (B) 0.9769 (C) 0.8781 (D) 0.8601

(3) The probability of 4 faults per 200 feet of such cable is: $f(4) = \frac{(0.6*2)^4}{4!} e^{-0.6*2}$

(A) 0.02602 (B) 0.1976 (C) 0.8024 (D) 0.9739