جامعة الملك سعود
قسم الإحصاء وبحوث العمليات
الإمتحان النهائي للمقرر 223 أحص
العام 1437 – 1438 الفصل الثاني

الأسم :

$\overline{7}$	السؤال 1
$\overline{2}$	السؤال 2
$\overline{6}$	السؤال 3
$\overline{6}$	السؤال 4
$\overline{6}$	السؤال 5
$\overline{8}$	السؤال 6
$\overline{5}$	السؤال 7
$\overline{40}$	المجموع

المحصلة	الإمتحان النهائي	الأعمال الفصلية
$\overline{100}$	$\overline{40}$	$\overline{60}$

Answer The Questions

Q1 (7M): Let $f(x; \theta) = \theta(1 - x)^{\theta-1}$; $0 < x < 1$, then:

1- Find $\hat{\theta}_{MLE}$:   (4M)

2- Find $\hat{\theta}_M$:   (3M)

Q2(2M): Complete the spaces..........:

$$\frac{\partial linL}{\partial \theta} = \frac{n}{(1-\theta)^2}\left[\left(\frac{\sum_{i=1}^{n} linx_i}{n}\right) - \left(\frac{\theta-1}{\theta}\right)\right] \Rightarrow \left(\widehat{\frac{\theta-1}{\theta}}\right)_{MLE} = ...$$

Q3(6M): Let $f(x; \theta)$ be some *pdf*, and if $\pi(\theta|\underline{x})$ is the posterior of θ.

Assume that S is *S.S*, and λ, a, b, σ are constants, then **Select two cases** among $\underline{A}, \underline{B}, \underline{C}$:

$\underline{A}$- $\pi(\theta|\underline{x}) = K(\underline{x}) \times \frac{\theta^n}{\Gamma n} s^{(n-1)} e^{-\theta S} \times \lambda e^{-\lambda\theta}; \quad \theta \geq 0$

$\underline{B}$- $\pi(\theta|\underline{x}) = K(\underline{x}) \binom{n}{S} \theta^S (1-\theta)^{n-S} \times \frac{1}{\beta(a,b)} \theta^{a-1}(1-\theta)^{b-1}; \quad 0 < \theta < 1$

$\underline{C}$- $\pi(\theta|\underline{x}) = K(\underline{x}) e^{-\frac{1}{2}\left[\frac{\theta - \left(S - \frac{\sigma^2\lambda}{n}\right)}{\sigma/\sqrt{n}}\right]^2}$

1- Discover what is the name of $\pi(\theta|\underline{x})$ (2M)

2- Compute T_B of θ (4M)

Q4 (6M) Let $f(x; \theta, \eta) = \theta e^{-\theta(x-\eta)};\ x > \eta$, then find:

$$(\hat{\eta}_M, \hat{\theta}_M) \ \text{Or} \ (\hat{\eta}_{MLE}, \hat{\theta}_{MLE})$$

Q5(6M): Let: $f(x; \theta) = \frac{2x}{\theta^2}$; $x \in (0, \theta)$, : then:

1-Find $\hat{\theta}_{MLE}$:   (3M)

2-Compute $E(X)$ and use it to find $\hat{\theta}_M$:   (3M)

Q6 (8M): Let $f(x; \theta_1, \theta_2) = \frac{1}{2\theta_2}$; $\theta_1 - \theta_2 \leq x \leq \theta_1 + \theta_2$, then
1- Find $(\widehat{\theta_1})_{MLE}, (\widehat{\theta_2})_{MLE}$.
2- Find $(\widehat{\theta_1})_M, (\widehat{\theta_2})_M$ given that $\mu = \theta_1$ & $\sigma^2 = \frac{1}{3}\theta_2^2$

5

Q7(5M). Let: $f(x; \theta) = \theta e^{-\theta x}$ and if $\hat{\theta}_{MLE} = 1/\bar{x}$ and if $\tau(\theta) = e^{-\theta}$, and if n is large, then:

a- What is $\widehat{\tau(\theta)}_{MLE} =$

b- What is the distribution of $\widehat{\tau(\theta)}_{MLE}$:

c- What is the expectation of $E\left[\widehat{\tau(\theta)}_{MLE}\right] =$

d- If $I = \frac{1}{\theta^2}$, then what is the variance $V\left[\widehat{\tau(\theta)}_{MLE}\right]$,

e- Estimate the 95% percentile point of the statistic $\widehat{\tau(\theta)}_{MLE}$: