

Student Name:

ID#:

Answer the following

Problem 1

Consider the following the linear regression model

$$\left. \begin{array}{l} Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad i = 1, 2, \dots, n, \\ E(\varepsilon_i) = 0, \quad \text{Var}(\varepsilon_i) = \sigma^2 \text{ and } \text{Cov}(\varepsilon_i, \varepsilon_j) = 0, \quad i \neq j \end{array} \right\}$$

- (a) Write the given regression model with the associated conditions in the matrix form.
- (b) Use the least square method to derive the estimate of the vector of parameters $\beta' = (\beta_0, \beta_1)$.

Problem 2

A marketing researcher studied annual sales of a product that had been introduced 10 years ago. The data are as follows, where X is the year (coded) and Y is the sales in thousands of units:

$i:$	1	2	3	4	5	6	7	8	9	10
$X_j:$	0	1	2	3	4	5	6	7	8	9
$Y_j:$	98	135	162	178	221	232	283	300	374	395

- Estimate the simple linear regression models before and after the transform $Y' = \sqrt{Y}$
- Compare between the two models based on the meaning of the coefficient of determination.
- Calculate 90% CI for E(Y) in when X=11 before and after the transform. Then, compare the results.

Problem 3

To investigate the simple linear model $Y = \beta_0 + \beta_1 X + \varepsilon$, we have the following data:

$$X'X = \begin{bmatrix} 60 & 3599 \\ 3599 & 224091 \end{bmatrix}, \quad X'Y = \begin{bmatrix} 5098 \\ 296024 \end{bmatrix} \quad \text{and} \quad Y'Y = 448662$$

- (a) Estimate the coefficients of the model.
- (b) Find the standard deviation of the coefficients.
- (c) Calculate R^2 and the correlation coefficient and interpret the results.
- (d) Find 90% prediction interval of Y_{New} when $X = 50$

