- Paired-sample t-test
- One-sample proportion
- Two-sample-proportion
- One-sample variance test
- Two-sample variances test

3.4) Paired-Sample t-test

To perform this test, select

Stat > Basic Statistics > Paired-Sample t-test

Use the Paired-sample t-test to compare between the means of paired observations taken from the same population. This can be very useful to see the effectiveness of a treatment on some objects.

For Paired-Sample test, the hypotheses are:

Null hypothesis

H_0: $\mu_d = \mu_1 - \mu_2 = 0$

Alternative hypothesis

Choose one:

$H_{1:}$ $\mu_d = \mu_1 - \mu_2 \neq 0$
H_1: H_d: $\mu_1 - \mu_2 < 0$
H_1: H_d: $\mu_1 - \mu_2 > 0$

In general, the test can be done through 4 steps, or p-values approach as:

Step-1: Setup the hypotheses

H_0: μ_d: $\mu_1 - \mu_2 = 0$ vs H_1: μ_d: $\mu_1 - \mu_2 \neq 0$ (or <0 , >0)

Step-2: **p-value = ? , then (reject H0 in p-value $\leq \alpha$)**

Example

An assertiveness training course has just been added to the services offered by a counseling center. To measure its effectiveness, ten students are given a test at the beginning of the course and again at the end. A high score on the test implies high assertiveness. Do the data provide sufficient evidence to conclude that people are more assertive after taking the course? α=.05

Before Course	After Course
50	65
62	68
51	52
41	43
63	60
56	70
49	48
67	69
42	53
57	61

Entering the data in Minitab:

Before	After	Difference
50	65	-15
62	68	-6
.	.	.
.	.	.
57	61	-4

Since the plotted points fall approximately along a straight line, we can conclude that the normal distribution assumption for the differences is reasonable. So, we can go ahead and conduct paired-t test.

Minitab Output:

Paired T for Before - After

	N	Mean	StDev	SE Mean
Before	10	53.80	8.75	2.77
After	10	58.90	9.46	2.99
Difference	10	-5.10	6.26	1.98

95% upper bound for mean difference: -1.47

T-Test of mean difference = 0 (vs < 0): T-Value = -2.58 P-Value = 0.015

Step-1: Setup the hypotheses

H_d: H_d: $\mu_1 - \mu_2 = 0$ vs H_d: H_d: $\mu_1 - \mu_2 < 0$

Step-2: P-Value = 0.015 **, then (reject H0 in p-value** $\leq \alpha = 0.05$**)**

Then, we have sufficient evidence to conclude that the population mean score for students before taking the course is less than the mean score after taking the course.

3.5) One-sample proportion

What is 1 proportion test?

To perform this test, select **Stat** > **Basic Statistics** > **1 Proportion**.

Use the 1 proportion test to estimate the proportion of a population and compare it to a target or reference value. Using this test, you can:

- Determine whether the proportion for a group differs from a specified value.
- Calculate a range of values that is likely to include the population proportion.

For example, you have data for a sample of customers and you wish to determine whether your placement rate is better than the published claims of a different employment agency.

Hypotheses for 1 proportion test

For 1 Proportion, the hypotheses are:

Null hypothesis

H_0: $p = p_0$	The population proportion (p) equals the hypothesized proportion (p_0).

Alternative hypothesis

Choose one:

H_1: $p \neq p_0$	The population proportion (p) differs from the hypothesized proportion (p_0).
H_1: $p > p_0$	The population proportion (p) is greater than the hypothesized proportion (ρ_0).
H_1: $p < p_0$	The population proportion (p) is less than the hypothesized proportion (p_0).

Example

A county district attorney would like to run for the office of state district attorney. She has decided that she will give up her county office and run for state office if more than 65% of her party constituents support her. You need to test H0: p = .65 versus H1: p > .65.

As her campaign manager, you collected data on 950 randomly selected party members and find that 560 party members support the candidate. A test of proportion was performed to determine whether or not the proportion of supporters was greater than the required proportion of 0.65. In addition, a 95% confidence bound was constructed to determine the lower bound for the proportion of supporters.

1 Choose **Stat > Basic Statistics > 1 Proportion**.

2 Choose **Summarized data**.

3 In **Number of events**, enter *560*. In **Number of trials**, enter *950*.

4 Check **Perform hypothesis test**. In **Hypothesized proportion**, enter *0.65*.

5 Click **Options**. Under **Alternative**, choose **greater than**. Click **OK** in each dialog box.

Session window output

Test and CI for One Proportion

Test of p = 0.65 vs p > 0.65

Sample	X	N	Sample p	95% Lower Bound	Exact P-Value
1	560	950	0.589474	0.562515	1.000

Interpreting the results

The p-value is 1.0 suggests that the data are consistent with the null hypothesis (H0: p = 0.65), that is, the proportion of party members that support the candidate is not greater than the required proportion of 0.65. As her campaign manager, you would advise her not to run for the office of state district attorney.

3-6) Two-sample proportion

To perform this test, select **Stat** > **Basic Statistics** > **2 Proportions**.

Use this analysis to:

- Determine whether the proportions of two groups differ
- Calculate a range of values that is likely to include the difference between the population proportions

For example, suppose you wanted to know whether the proportion of consumers who return a survey could be increased by providing an incentive such as a product sample. You might include the product sample with half of your mailings and determine whether you have more responses from the group that received the sample than from those who did not.

Hypotheses for 2 proportions test

For 2 Proportions, the hypotheses are:

Null hypothesis

H_0: $p_1 - p_2 = d_0$	The difference between the population proportions ($p_1 - p_2$) equals the hypothesized difference (d_0).

Alternative hypothesis

Choose one:

H_1: $p_1 - p_2 \neq d_0$	The difference between the population proportions ($p_1 - p_2$) does not equal the hypothesized difference (d_0).
H_1: $p_1 - p_2 > d_0$	The difference between the population proportions ($p_1 - p_2$) is greater than the hypothesized difference (d_0).
H_1: $p_1 - p_2 < d_0$	The difference between the population proportions ($p_1 - p_2$) is less than the hypothesized difference (d_0).

Example

As your corporation's purchasing manager, you need to authorize the purchase of twenty new photocopy machines. After comparing many brands in terms of price, copy quality, warranty, and features, you have narrowed the choice to two: Brand X and Brand Y. You decide that the determining factor will be the reliability of the brands as defined by the proportion requiring service within one year of purchase.

Because your corporation already uses both of these brands, you were able to obtain information on the service history of 50 randomly selected machines of each brand. Records indicate that six Brand X machines and eight Brand Y machines needed service. Use this information to guide your choice of brand for purchase.

1 Choose **Stat > Basic Statistics > 2 Proportions**.

2 Choose **Summarized data**.

3 In **First sample**, under **Events**, enter *44*. Under **Trials**, enter *50*.

4 In **Second sample**, under **Events**, enter *42*. Under **Trials**, enter *50*. Click **OK**.

Session window output

Test and CI for Two Proportions

Sample	X	N	Sample p
1	44	50	0.880000
2	42	50	0.840000

Difference = p (1) - p (2)

Estimate for difference: 0.04

95% CI for difference: (-0.0957903, 0.175790)

Test for difference = 0 (vs not = 0): Z = 0.58 P-Value = 0.564

From the results, we can write

(1) H_0: p_1- p_2 = 0 vs H_1: p_1- p_2 ≠ 0

(2) **P-value-0.564 > .05 → cannot reject H0, this means there is no significance difference**

Therefore, the data are consistent with the null hypothesis that the population proportions are equal. In other words, the proportion of photocopy machines that needed service in the first year did not differ depending on brand.

3-7) One-sample variance test

To perform this test, select **Stat** > **Basic Statistics** > **1 Variance**.

Use the 1 Variance test to estimate the variance or the standard deviation of a population and compare it to a target or reference value. Using this test, you can:

- Determine whether the variance or the standard deviation of a group differs from a specified value.
- Calculate a range of values that is likely to include the population variance or standard deviation.

For example, you receive a shipment of unprocessed lumber and you want to assess whether the moisture content is too variable.

For 1 Variance, the hypotheses are:

Null hypothesis

H_0: $\sigma = \sigma_0$	The population standard deviation (σ) equals the hypothesized standard deviation (σ_0).

Alternative hypothesis

Choose one:

H_1: $\sigma \neq \sigma_0$	The population standard deviation (σ) differs from the hypothesized standard deviation (σ_0).
H_1: $\sigma > \sigma_0$	The population standard deviation (σ) is greater than the hypothesized standard deviation (σ_0).

NOTE
If you are testing the variance, substitute variance (σ^2) for standard deviation (σ) in the hypotheses.

Example

You are a quality control inspector at a factory that builds high precision parts for aircraft engines, including a metal pin that must measure 15 inches in length. Safety laws dictate that the variance of the pins' length must not exceed $0.001 in^2$. Previous analyses determined that pin length is normally distributed. You collect a sample of 100 pins and measure their length in order to conduct a hypothesis test and create a confidence interval for the population variance.

1 Open the worksheet AIRPLANEPIN.MTW.

2 Choose **Stat > Basic Statistics > 1 Variance**.

3 Under **Data**, choose **Samples in columns**.

4 In **Columns**, enter '*Pin length* '.

5 Check **Perform hypothesis test** and choose **Hypothesized variance**.

6 In **Value**, enter *0.001*.

7 Click **Options**. Under **Alternative**, choose **less than**.

8 Click **OK** in each dialog box.

Session window output

Test and CI for One Variance: Pin length

Method

Null hypothesis Sigma-squared = 0.001

Alternative hypothesis Sigma-squared < 0.001

Variable	N	StDev	Variance
Pin length	100	0.0267	0.000715

95% One-Sided Confidence Intervals

Variable	Method	Upper Bound for StDev	Upper Bound for Variance
Pin length	Chi-Square	0.0303	0.000919

Variable	Method	Statistic	DF	P-Value
Pin length	Chi-Square	70.77	99	0.014

From the results, we can write

(1) **H_0: σ =0.001 VS H_0: σ < 0.001**

(2) **P-value=0.014 < 0.05**

We reject H0.

Interpreting the results

Because the data comes from a normally distributed population, refer to the chi-square method. The p-value for the one-sided hypothesis test is 0.014. This value is sufficiently low to reject the null hypothesis and conclude that the variance of pin length is less than 0.001. You can further hone your estimate of the population variance by considering the 95% upper bound, which provides a value that the population variance is likely to be below. From this analysis, you should conclude that the variance of pin length is small enough to meet specifications and ensure passenger safety.

3-8) Two-sample variances test

To perform this test, select **Stat** > **Basic Statistics** > **2 Variances**.

Use this analysis to:

- Determine whether the variances or standard deviations of two groups differ.
- Calculate a range of values that is likely to include the population ratio of the variances or standard deviations of the two groups.

For example, suppose managers of a car manufacturer are deciding whether to switch to a new camshaft vendor (Vendor A) that claims they produce camshafts with less variance in diameter than the current vendor (Vendor B). Safety regulations set tight specification limits for camshaft diameter; therefore, the managers should choose the vendor with less variance in camshaft diameter.

This test is useful for verifying assumptions of equal variance, which is the foundation of some statistical procedures. It is also useful for quality improvement situations: you can use this test to compare variance within subgroups to variance between subgroups. Also, you can use this test to compare process variance before and after implementation of a quality improvement program.

For 2 Variances, the hypotheses are:

Null hypothesis

$H_0: \sigma_1 / \sigma_2 = K$	The ratio between the first population standard deviation (σ_1) and the second population standard deviation (σ_2) is equal to the hypothesized ratio (K).

Alternative hypothesis

Choose one:

H_1: $\sigma_1 / \sigma_2 \neq K$	The ratio between the first population standard deviation (σ_1)and the second population standard deviation (σ_2) does not equal the hypothesized ratio (K).
H_1: $\sigma_1 / \sigma_2 > K$	The ratio between the first population standard deviation (σ_1)and the second population standard deviation (σ_2) is greater than the hypothesized ratio (K).
H_1: $\sigma_1 / \sigma_2 > K$	The ratio between the first population standard deviation (σ_1)and the second population standard deviation (σ_2) is less than the hypothesized ratio (K).

NOTE

If you are testing the ratio of variances, substitute variance (σ^2) for standard deviation (σ) in the hypotheses.

Example

A study was performed in order to evaluate the effectiveness of two devices for improving the efficiency of gas home-heating systems. Energy consumption in houses was measured after one of the two devices was installed. The two devices were an electric vent damper (Damper = 1) and a thermally activated vent damper (Damper = 2). The energy consumption data (BTU.In) are stacked in one column with a grouping column (Damper) containing identifiers or subscripts to denote the population. You are interested in comparing the standard deviations of the two populations so that you can construct a 2-Sample t-test and confidence interval to compare the two dampers.

1 Open the worksheet FURNACE.MTW.

2 Choose **Stat > Basic Statistics > 2 Variances**.

3 Under **Data**, choose **Samples in one column**.

4 In **Samples**, enter '*BTU.In*'.

5 In **Subscripts**, enter *Damper*. Click **OK**.

Session window output

Test and CI for Two Variances: BTU.In vs Damper

Test and CI for Two Variances: BTU.In vs Damper

```
Method

Null hypothesis         Sigma(1) / Sigma(2) = 1
Alternative hypothesis  Sigma(1) / Sigma(2) not = 1
Significance level      Alpha = 0.05


Statistics

Damper   N  StDev  Variance
1       40  3.020     9.120
2       50  2.767     7.656

Ratio of standard deviations = 1.091
Ratio of variances = 1.191


95% Confidence Intervals

                                  CI for
Distribution   CI for StDev      Variance
of Data           Ratio            Ratio
Normal        (0.812, 1.483)  (0.659, 2.199)
Continuous    (0.697, 1.412)  (0.486, 1.992)


Tests

                                                 Test
Method                          DF1  DF2  Statistic  P-Value
F Test (normal)                  39   49       1.19    0.558
Levene's Test (any continuous)    1   88       0.00    0.996
```

From the results, we see that

(1) $H_0: \sigma_1 / \sigma_2 = 1$ vs $H_1: \sigma_1 / \sigma_2 \neq 1$

(2) P-value= 0.558, then cannot reject H0, this means that the variance of Damper = 1 is equal of Damper = 2.