

$$A = \left\{ \frac{m}{2m+3n}, m, n \in \mathbb{N} \right\} = \left\{ \frac{1}{\frac{2m+3n}{m}} \right\} = \left\{ \frac{1}{\left\{ \begin{array}{l} 2+3\frac{n}{m} \\ m \end{array} \right\}} \right\} \begin{array}{l} \rightarrow \text{sup} = \infty \\ \rightarrow \text{inf} = 2 \end{array}$$

$$\Rightarrow \inf(A) = 0, \text{sup}(A) = \frac{1}{2}$$

Proof: $\text{sup}(A) = \frac{1}{2}$: Assume u is an upper bound of A . (want: $u > \frac{1}{2}$?)

Assume $u < \frac{1}{2} \Rightarrow 0 < \frac{1}{2} - u \Rightarrow \exists M \in \mathbb{N}$ such that: $\frac{1}{M} < \frac{1}{2} - u$

$$\Rightarrow \frac{1}{2} - \frac{M}{2M+3} = \frac{2M+3-2M}{4M+6} = \frac{3}{4M+6} < \frac{3}{4M} < \frac{1}{M} < \frac{1}{2} - u$$

$\Rightarrow \frac{-M}{2M+3} < -u \Rightarrow \exists \frac{M}{2M+3} > u \Rightarrow$ Contradiction with u upper bound of A .

Inf(A) = 0: Assume v is lower bound of A . (want $v \leq 0$?)

Assume $v > 0 \Rightarrow \exists N \in \mathbb{N}$ such that $\frac{1}{N} < v$

$$\Rightarrow A \ni \frac{1}{2+3N} < \frac{1}{3N} < \frac{1}{N} < v$$

\Rightarrow contradiction with v lower bound of A