لا يكتب في
هذا الهامش

$$E=\left\{\frac{n^2+9m^4+18m^2n}{3m^2n}\ ;\ m,n\in\mathbb{N}\right\}$$

$\inf E$: $E=\left\{\frac{n^2}{3m^2n}+\frac{9m^4}{3m^2n}+\frac{18m^2n}{3m^2n}\ ,n,m\in\mathbb{N}\right\}$

$$=\left\{\frac{n}{3m^2}+\frac{3m^2}{n}+6\ ,m,n\in\mathbb{N}\right\}$$

Use $\sqrt{ab}\le\frac{a+b}{2}$. Take $a=\frac{n}{3m^2}$, $b=\frac{3m^2}{n}$

$$\sqrt{\frac{n}{3m^2}\cdot\frac{3m^2}{n}}\le\frac{\frac{n}{3m^2}+\frac{3m^2}{n}}{2}$$

$$2\le\frac{n}{3m^2}+\frac{3m^2}{n}$$

$$8=2+6\le\frac{n}{3m^2}+\frac{3m^2}{n}+6$$

What are n,m so that $\frac{n}{3m^2}+\frac{3m^2}{n}=2$??

$$\frac{9m^4+n^2}{3m^2n}=2\Rightarrow 9m^4-6m^2n+n^2=0$$

$$(3m^2-n)^2=0\Rightarrow 3m^2=n$$

Take $n=3, m=1$

$\Rightarrow 8\in E\Rightarrow 8=\min E=\inf E.$

$\sup E$: Put $n=1\to\frac{1+9m^4+18m^2}{3m^2}$

as $m\to\infty\Rightarrow\frac{1+9m^4+18m^2}{3m^2}\to\infty$

$\Rightarrow E$ is not bounded above

$\Rightarrow \sup(E)=\infty$

$\max(E)$ D.N.E