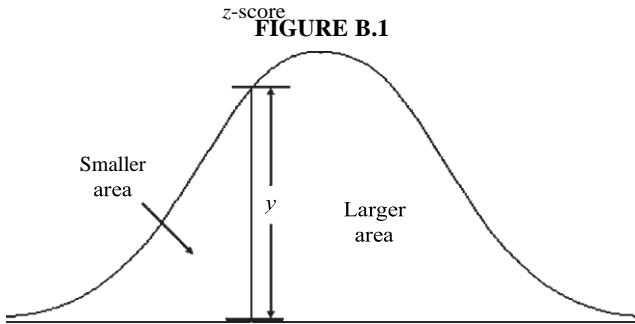


**CRITICAL VALUE TABLES**



**TABLE B.1 The Normal Distribution.**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 0.00    | 0.5000       | 0.5000      | 0.3989 |
| 0.01    | 0.4960       | 0.5040      | 0.3989 |
| 0.02    | 0.4920       | 0.5080      | 0.3989 |
| 0.03    | 0.4880       | 0.5120      | 0.3988 |
| 0.04    | 0.4840       | 0.5160      | 0.3986 |
| 0.05    | 0.4801       | 0.5199      | 0.3984 |
| 0.06    | 0.4761       | 0.5239      | 0.3982 |
| 0.07    | 0.4721       | 0.5279      | 0.3980 |
| 0.08    | 0.4681       | 0.5319      | 0.3977 |
| 0.09    | 0.4641       | 0.5359      | 0.3973 |
| 0.10    | 0.4602       | 0.5398      | 0.3970 |
| 0.11    | 0.4562       | 0.5438      | 0.3965 |
| 0.12    | 0.4522       | 0.5478      | 0.3961 |
| 0.13    | 0.4483       | 0.5517      | 0.3956 |
| 0.14    | 0.4443       | 0.5557      | 0.3951 |
| 0.15    | 0.4404       | 0.5596      | 0.3945 |

*(Continued)*

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 0.16    | 0.4364       | 0.5636      | 0.3939 |
| 0.17    | 0.4325       | 0.5675      | 0.3932 |
| 0.18    | 0.4286       | 0.5714      | 0.3925 |
| 0.19    | 0.4247       | 0.5753      | 0.3918 |
| 0.20    | 0.4207       | 0.5793      | 0.3910 |
| 0.21    | 0.4168       | 0.5832      | 0.3902 |
| 0.22    | 0.4129       | 0.5871      | 0.3894 |
| 0.23    | 0.4090       | 0.5910      | 0.3885 |
| 0.24    | 0.4052       | 0.5948      | 0.3876 |
| 0.25    | 0.4013       | 0.5987      | 0.3867 |
| 0.26    | 0.3974       | 0.6026      | 0.3857 |
| 0.27    | 0.3936       | 0.6064      | 0.3847 |
| 0.28    | 0.3897       | 0.6103      | 0.3836 |
| 0.29    | 0.3859       | 0.6141      | 0.3825 |
| 0.30    | 0.3821       | 0.6179      | 0.3814 |
| 0.31    | 0.3783       | 0.6217      | 0.3802 |
| 0.32    | 0.3745       | 0.6255      | 0.3790 |
| 0.33    | 0.3707       | 0.6293      | 0.3778 |
| 0.34    | 0.3669       | 0.6331      | 0.3765 |
| 0.35    | 0.3632       | 0.6368      | 0.3752 |
| 0.36    | 0.3594       | 0.6406      | 0.3739 |
| 0.37    | 0.3557       | 0.6443      | 0.3725 |
| 0.38    | 0.3520       | 0.6480      | 0.3712 |
| 0.39    | 0.3483       | 0.6517      | 0.3697 |
| 0.40    | 0.3446       | 0.6554      | 0.3683 |
| 0.41    | 0.3409       | 0.6591      | 0.3668 |
| 0.42    | 0.3372       | 0.6628      | 0.3653 |
| 0.43    | 0.3336       | 0.6664      | 0.3637 |
| 0.44    | 0.3300       | 0.6700      | 0.3621 |
| 0.45    | 0.3264       | 0.6736      | 0.3605 |
| 0.46    | 0.3228       | 0.6772      | 0.3589 |
| 0.47    | 0.3192       | 0.6808      | 0.3572 |
| 0.48    | 0.3156       | 0.6844      | 0.3555 |
| 0.49    | 0.3121       | 0.6879      | 0.3538 |
| 0.50    | 0.3085       | 0.6915      | 0.3521 |
| 0.51    | 0.3050       | 0.6950      | 0.3503 |
| 0.52    | 0.3015       | 0.6985      | 0.3485 |
| 0.53    | 0.2981       | 0.7019      | 0.3467 |
| 0.54    | 0.2946       | 0.7054      | 0.3448 |
| 0.55    | 0.2912       | 0.7088      | 0.3429 |
| 0.56    | 0.2877       | 0.7123      | 0.3410 |
| 0.57    | 0.2843       | 0.7157      | 0.3391 |
| 0.58    | 0.2810       | 0.7190      | 0.3372 |

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 0.59    | 0.2776       | 0.7224      | 0.3352 |
| 0.60    | 0.2743       | 0.7257      | 0.3332 |
| 0.61    | 0.2709       | 0.7291      | 0.3312 |
| 0.62    | 0.2676       | 0.7324      | 0.3292 |
| 0.63    | 0.2643       | 0.7357      | 0.3271 |
| 0.64    | 0.2611       | 0.7389      | 0.3251 |
| 0.65    | 0.2578       | 0.7422      | 0.3230 |
| 0.66    | 0.2546       | 0.7454      | 0.3209 |
| 0.67    | 0.2514       | 0.7486      | 0.3187 |
| 0.68    | 0.2483       | 0.7517      | 0.3166 |
| 0.69    | 0.2451       | 0.7549      | 0.3144 |
| 0.70    | 0.2420       | 0.7580      | 0.3123 |
| 0.71    | 0.2389       | 0.7611      | 0.3101 |
| 0.72    | 0.2358       | 0.7642      | 0.3079 |
| 0.73    | 0.2327       | 0.7673      | 0.3056 |
| 0.74    | 0.2296       | 0.7704      | 0.3034 |
| 0.75    | 0.2266       | 0.7734      | 0.3011 |
| 0.76    | 0.2236       | 0.7764      | 0.2989 |
| 0.77    | 0.2206       | 0.7794      | 0.2966 |
| 0.78    | 0.2177       | 0.7823      | 0.2943 |
| 0.79    | 0.2148       | 0.7852      | 0.2920 |
| 0.80    | 0.2119       | 0.7881      | 0.2897 |
| 0.81    | 0.2090       | 0.7910      | 0.2874 |
| 0.82    | 0.2061       | 0.7939      | 0.2850 |
| 0.83    | 0.2033       | 0.7967      | 0.2827 |
| 0.84    | 0.2005       | 0.7995      | 0.2803 |
| 0.85    | 0.1977       | 0.8023      | 0.2780 |
| 0.86    | 0.1949       | 0.8051      | 0.2756 |
| 0.87    | 0.1922       | 0.8078      | 0.2732 |
| 0.88    | 0.1894       | 0.8106      | 0.2709 |
| 0.89    | 0.1867       | 0.8133      | 0.2685 |
| 0.90    | 0.1841       | 0.8159      | 0.2661 |
| 0.91    | 0.1814       | 0.8186      | 0.2637 |
| 0.92    | 0.1788       | 0.8212      | 0.2613 |
| 0.93    | 0.1762       | 0.8238      | 0.2589 |
| 0.94    | 0.1736       | 0.8264      | 0.2565 |
| 0.95    | 0.1711       | 0.8289      | 0.2541 |
| 0.96    | 0.1685       | 0.8315      | 0.2516 |
| 0.97    | 0.1660       | 0.8340      | 0.2492 |
| 0.98    | 0.1635       | 0.8365      | 0.2468 |
| 0.99    | 0.1611       | 0.8389      | 0.2444 |
| 1.00    | 0.1587       | 0.8413      | 0.2420 |

*(Continued)*

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 1.01    | 0.1562       | 0.8438      | 0.2396 |
| 1.02    | 0.1539       | 0.8461      | 0.2371 |
| 1.03    | 0.1515       | 0.8485      | 0.2347 |
| 1.04    | 0.1492       | 0.8508      | 0.2323 |
| 1.05    | 0.1469       | 0.8531      | 0.2299 |
| 1.06    | 0.1446       | 0.8554      | 0.2275 |
| 1.07    | 0.1423       | 0.8577      | 0.2251 |
| 1.08    | 0.1401       | 0.8599      | 0.2227 |
| 1.09    | 0.1379       | 0.8621      | 0.2203 |
| 1.10    | 0.1357       | 0.8643      | 0.2179 |
| 1.11    | 0.1335       | 0.8665      | 0.2155 |
| 1.12    | 0.1314       | 0.8686      | 0.2131 |
| 1.13    | 0.1292       | 0.8708      | 0.2107 |
| 1.14    | 0.1271       | 0.8729      | 0.2083 |
| 1.15    | 0.1251       | 0.8749      | 0.2059 |
| 1.16    | 0.1230       | 0.8770      | 0.2036 |
| 1.17    | 0.1210       | 0.8790      | 0.2012 |
| 1.18    | 0.1190       | 0.8810      | 0.1989 |
| 1.19    | 0.1170       | 0.8830      | 0.1965 |
| 1.20    | 0.1151       | 0.8849      | 0.1942 |
| 1.21    | 0.1131       | 0.8869      | 0.1919 |
| 1.22    | 0.1112       | 0.8888      | 0.1895 |
| 1.23    | 0.1093       | 0.8907      | 0.1872 |
| 1.24    | 0.1075       | 0.8925      | 0.1849 |
| 1.25    | 0.1056       | 0.8944      | 0.1826 |
| 1.26    | 0.1038       | 0.8962      | 0.1804 |
| 1.27    | 0.1020       | 0.8980      | 0.1781 |
| 1.28    | 0.1003       | 0.8997      | 0.1758 |
| 1.29    | 0.0985       | 0.9015      | 0.1736 |
| 1.30    | 0.0968       | 0.9032      | 0.1714 |
| 1.31    | 0.0951       | 0.9049      | 0.1691 |
| 1.32    | 0.0934       | 0.9066      | 0.1669 |
| 1.33    | 0.0918       | 0.9082      | 0.1647 |
| 1.34    | 0.0901       | 0.9099      | 0.1626 |
| 1.35    | 0.0885       | 0.9115      | 0.1604 |
| 1.36    | 0.0869       | 0.9131      | 0.1582 |
| 1.37    | 0.0853       | 0.9147      | 0.1561 |
| 1.38    | 0.0838       | 0.9162      | 0.1539 |
| 1.39    | 0.0823       | 0.9177      | 0.1518 |
| 1.40    | 0.0808       | 0.9192      | 0.1497 |
| 1.41    | 0.0793       | 0.9207      | 0.1476 |
| 1.42    | 0.0778       | 0.9222      | 0.1456 |
| 1.43    | 0.0764       | 0.9236      | 0.1435 |

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 1.44    | 0.0749       | 0.9251      | 0.1415 |
| 1.45    | 0.0735       | 0.9265      | 0.1394 |
| 1.46    | 0.0721       | 0.9279      | 0.1374 |
| 1.47    | 0.0708       | 0.9292      | 0.1354 |
| 1.48    | 0.0694       | 0.9306      | 0.1334 |
| 1.49    | 0.0681       | 0.9319      | 0.1315 |
| 1.50    | 0.0668       | 0.9332      | 0.1295 |
| 1.51    | 0.0655       | 0.9345      | 0.1276 |
| 1.52    | 0.0643       | 0.9357      | 0.1257 |
| 1.53    | 0.0630       | 0.9370      | 0.1238 |
| 1.54    | 0.0618       | 0.9382      | 0.1219 |
| 1.55    | 0.0606       | 0.9394      | 0.1200 |
| 1.56    | 0.0594       | 0.9406      | 0.1182 |
| 1.57    | 0.0582       | 0.9418      | 0.1163 |
| 1.58    | 0.0571       | 0.9429      | 0.1145 |
| 1.59    | 0.0559       | 0.9441      | 0.1127 |
| 1.60    | 0.0548       | 0.9452      | 0.1109 |
| 1.61    | 0.0537       | 0.9463      | 0.1092 |
| 1.62    | 0.0526       | 0.9474      | 0.1074 |
| 1.63    | 0.0516       | 0.9484      | 0.1057 |
| 1.64    | 0.0505       | 0.9495      | 0.1040 |
| 1.65    | 0.0495       | 0.9505      | 0.1023 |
| 1.66    | 0.0485       | 0.9515      | 0.1006 |
| 1.67    | 0.0475       | 0.9525      | 0.0989 |
| 1.68    | 0.0465       | 0.9535      | 0.0973 |
| 1.69    | 0.0455       | 0.9545      | 0.0957 |
| 1.70    | 0.0446       | 0.9554      | 0.0940 |
| 1.71    | 0.0436       | 0.9564      | 0.0925 |
| 1.72    | 0.0427       | 0.9573      | 0.0909 |
| 1.73    | 0.0418       | 0.9582      | 0.0893 |
| 1.74    | 0.0409       | 0.9591      | 0.0878 |
| 1.75    | 0.0401       | 0.9599      | 0.0863 |
| 1.76    | 0.0392       | 0.9608      | 0.0848 |
| 1.77    | 0.0384       | 0.9616      | 0.0833 |
| 1.78    | 0.0375       | 0.9625      | 0.0818 |
| 1.79    | 0.0367       | 0.9633      | 0.0804 |
| 1.80    | 0.0359       | 0.9641      | 0.0790 |
| 1.81    | 0.0351       | 0.9649      | 0.0775 |
| 1.82    | 0.0344       | 0.9656      | 0.0761 |
| 1.83    | 0.0336       | 0.9664      | 0.0748 |
| 1.84    | 0.0329       | 0.9671      | 0.0734 |
| 1.85    | 0.0322       | 0.9678      | 0.0721 |

*(Continued)*

**TABLE B.1** (*Continued*)

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 1.86    | 0.0314       | 0.9686      | 0.0707 |
| 1.87    | 0.0307       | 0.9693      | 0.0694 |
| 1.88    | 0.0301       | 0.9699      | 0.0681 |
| 1.89    | 0.0294       | 0.9706      | 0.0669 |
| 1.90    | 0.0287       | 0.9713      | 0.0656 |
| 1.91    | 0.0281       | 0.9719      | 0.0644 |
| 1.92    | 0.0274       | 0.9726      | 0.0632 |
| 1.93    | 0.0268       | 0.9732      | 0.0620 |
| 1.94    | 0.0262       | 0.9738      | 0.0608 |
| 1.95    | 0.0256       | 0.9744      | 0.0596 |
| 1.96    | 0.0250       | 0.9750      | 0.0584 |
| 1.97    | 0.0244       | 0.9756      | 0.0573 |
| 1.98    | 0.0239       | 0.9761      | 0.0562 |
| 1.99    | 0.0233       | 0.9767      | 0.0551 |
| 2.00    | 0.0228       | 0.9772      | 0.0540 |
| 2.01    | 0.0222       | 0.9778      | 0.0529 |
| 2.02    | 0.0217       | 0.9783      | 0.0519 |
| 2.03    | 0.0212       | 0.9788      | 0.0508 |
| 2.04    | 0.0207       | 0.9793      | 0.0498 |
| 2.05    | 0.0202       | 0.9798      | 0.0488 |
| 2.06    | 0.0197       | 0.9803      | 0.0478 |
| 2.07    | 0.0192       | 0.9808      | 0.0468 |
| 2.08    | 0.0188       | 0.9812      | 0.0459 |
| 2.09    | 0.0183       | 0.9817      | 0.0449 |
| 2.10    | 0.0179       | 0.9821      | 0.0440 |
| 2.11    | 0.0174       | 0.9826      | 0.0431 |
| 2.12    | 0.0170       | 0.9830      | 0.0422 |
| 2.13    | 0.0166       | 0.9834      | 0.0413 |
| 2.14    | 0.0162       | 0.9838      | 0.0404 |
| 2.15    | 0.0158       | 0.9842      | 0.0396 |
| 2.16    | 0.0154       | 0.9846      | 0.0387 |
| 2.17    | 0.0150       | 0.9850      | 0.0379 |
| 2.18    | 0.0146       | 0.9854      | 0.0371 |
| 2.19    | 0.0143       | 0.9857      | 0.0363 |
| 2.20    | 0.0139       | 0.9861      | 0.0355 |
| 2.21    | 0.0136       | 0.9864      | 0.0347 |
| 2.22    | 0.0132       | 0.9868      | 0.0339 |
| 2.23    | 0.0129       | 0.9871      | 0.0332 |
| 2.24    | 0.0125       | 0.9875      | 0.0325 |
| 2.25    | 0.0122       | 0.9878      | 0.0317 |
| 2.26    | 0.0119       | 0.9881      | 0.0310 |
| 2.27    | 0.0116       | 0.9884      | 0.0303 |
| 2.28    | 0.0113       | 0.9887      | 0.0297 |

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 2.29    | 0.0110       | 0.9890      | 0.0290 |
| 2.30    | 0.0107       | 0.9893      | 0.0283 |
| 2.31    | 0.0104       | 0.9896      | 0.0277 |
| 2.32    | 0.0102       | 0.9898      | 0.0270 |
| 2.33    | 0.0099       | 0.9901      | 0.0264 |
| 2.34    | 0.0096       | 0.9904      | 0.0258 |
| 2.35    | 0.0094       | 0.9906      | 0.0252 |
| 2.36    | 0.0091       | 0.9909      | 0.0246 |
| 2.37    | 0.0089       | 0.9911      | 0.0241 |
| 2.38    | 0.0087       | 0.9913      | 0.0235 |
| 2.39    | 0.0084       | 0.9916      | 0.0229 |
| 2.40    | 0.0082       | 0.9918      | 0.0224 |
| 2.41    | 0.0080       | 0.9920      | 0.0219 |
| 2.42    | 0.0078       | 0.9922      | 0.0213 |
| 2.43    | 0.0075       | 0.9925      | 0.0208 |
| 2.44    | 0.0073       | 0.9927      | 0.0203 |
| 2.45    | 0.0071       | 0.9929      | 0.0198 |
| 2.46    | 0.0069       | 0.9931      | 0.0194 |
| 2.47    | 0.0068       | 0.9932      | 0.0189 |
| 2.48    | 0.0066       | 0.9934      | 0.0184 |
| 2.49    | 0.0064       | 0.9936      | 0.0180 |
| 2.50    | 0.0062       | 0.9938      | 0.0175 |
| 2.51    | 0.0060       | 0.9940      | 0.0171 |
| 2.52    | 0.0059       | 0.9941      | 0.0167 |
| 2.53    | 0.0057       | 0.9943      | 0.0163 |
| 2.54    | 0.0055       | 0.9945      | 0.0158 |
| 2.55    | 0.0054       | 0.9946      | 0.0154 |
| 2.56    | 0.0052       | 0.9948      | 0.0151 |
| 2.57    | 0.0051       | 0.9949      | 0.0147 |
| 2.58    | 0.0049       | 0.9951      | 0.0143 |
| 2.59    | 0.0048       | 0.9952      | 0.0139 |
| 2.60    | 0.0047       | 0.9953      | 0.0136 |
| 2.61    | 0.0045       | 0.9955      | 0.0132 |
| 2.62    | 0.0044       | 0.9956      | 0.0129 |
| 2.63    | 0.0043       | 0.9957      | 0.0126 |
| 2.64    | 0.0041       | 0.9959      | 0.0122 |
| 2.65    | 0.0040       | 0.9960      | 0.0119 |
| 2.66    | 0.0039       | 0.9961      | 0.0116 |
| 2.67    | 0.0038       | 0.9962      | 0.0113 |
| 2.68    | 0.0037       | 0.9963      | 0.0110 |
| 2.69    | 0.0036       | 0.9964      | 0.0107 |
| 2.70    | 0.0035       | 0.9965      | 0.0104 |

*(Continued)*

**TABLE B.1 (Continued)**

| z-score | Smaller area | Larger area | y      |
|---------|--------------|-------------|--------|
| 2.71    | 0.0034       | 0.9966      | 0.0101 |
| 2.72    | 0.0033       | 0.9967      | 0.0099 |
| 2.73    | 0.0032       | 0.9968      | 0.0096 |
| 2.74    | 0.0031       | 0.9969      | 0.0093 |
| 2.75    | 0.0030       | 0.9970      | 0.0091 |
| 2.76    | 0.0029       | 0.9971      | 0.0088 |
| 2.77    | 0.0028       | 0.9972      | 0.0086 |
| 2.78    | 0.0027       | 0.9973      | 0.0084 |
| 2.79    | 0.0026       | 0.9974      | 0.0081 |
| 2.80    | 0.0026       | 0.9974      | 0.0079 |
| 2.81    | 0.0025       | 0.9975      | 0.0077 |
| 2.82    | 0.0024       | 0.9976      | 0.0075 |
| 2.83    | 0.0023       | 0.9977      | 0.0073 |
| 2.84    | 0.0023       | 0.9977      | 0.0071 |
| 2.85    | 0.0022       | 0.9978      | 0.0069 |
| 2.86    | 0.0021       | 0.9979      | 0.0067 |
| 2.87    | 0.0021       | 0.9979      | 0.0065 |
| 2.88    | 0.0020       | 0.9980      | 0.0063 |
| 2.89    | 0.0019       | 0.9981      | 0.0061 |
| 2.90    | 0.0019       | 0.9981      | 0.0060 |
| 2.91    | 0.0018       | 0.9982      | 0.0058 |
| 2.92    | 0.0018       | 0.9982      | 0.0056 |
| 2.93    | 0.0017       | 0.9983      | 0.0055 |
| 2.94    | 0.0016       | 0.9984      | 0.0053 |
| 2.95    | 0.0016       | 0.9984      | 0.0051 |
| 2.96    | 0.0015       | 0.9985      | 0.0050 |
| 2.97    | 0.0015       | 0.9985      | 0.0048 |
| 2.98    | 0.0014       | 0.9986      | 0.0047 |
| 2.99    | 0.0014       | 0.9986      | 0.0046 |
| 3.00    | 0.0013       | 0.9987      | 0.0044 |
| 3.10    | 0.0010       | 0.9990      | 0.0033 |
| 3.20    | 0.0007       | 0.9993      | 0.0024 |
| 3.30    | 0.0005       | 0.9995      | 0.0017 |
| 3.50    | 0.0002       | 0.9998      | 0.0009 |
| 3.75    | 0.0001       | 0.9999      | 0.0004 |
| 4.00    | 0.0000       | 1.0000      | 0.0001 |

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Source: Adapted by the authors from Hastings (1955).



**TABLE B.2 The  $\chi^2$  Distribution.**

| <i>df</i> | 0.99  | 0.975 | 0.95  | 0.9   | 0.1   | 0.05  | 0.025 | 0.01  |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1         | 0.00  | 0.00  | 0.00  | 0.02  | 2.71  | 3.84  | 5.02  | 6.63  |
| 2         | 0.02  | 0.05  | 0.10  | 0.21  | 4.61  | 5.99  | 7.38  | 9.21  |
| 3         | 0.11  | 0.22  | 0.35  | 0.58  | 6.25  | 7.81  | 9.35  | 11.34 |
| 4         | 0.30  | 0.48  | 0.71  | 1.06  | 7.78  | 9.49  | 11.14 | 13.28 |
| 5         | 0.55  | 0.83  | 1.15  | 1.61  | 9.24  | 11.07 | 12.83 | 15.09 |
| 6         | 0.87  | 1.24  | 1.64  | 2.20  | 10.64 | 12.59 | 14.45 | 16.81 |
| 7         | 1.24  | 1.69  | 2.17  | 2.83  | 12.02 | 14.07 | 16.01 | 18.48 |
| 8         | 1.65  | 2.18  | 2.73  | 3.49  | 13.36 | 15.51 | 17.53 | 20.09 |
| 9         | 2.09  | 2.70  | 3.33  | 4.17  | 14.68 | 16.92 | 19.02 | 21.67 |
| 10        | 2.56  | 3.25  | 3.94  | 4.87  | 15.99 | 18.31 | 20.48 | 23.21 |
| 11        | 3.05  | 3.82  | 4.57  | 5.58  | 17.28 | 19.68 | 21.92 | 24.73 |
| 12        | 3.57  | 4.40  | 5.23  | 6.30  | 18.55 | 21.03 | 23.34 | 26.22 |
| 13        | 4.11  | 5.01  | 5.89  | 7.04  | 19.81 | 22.36 | 24.74 | 27.69 |
| 14        | 4.66  | 5.63  | 6.57  | 7.79  | 21.06 | 23.68 | 26.12 | 29.14 |
| 15        | 5.23  | 6.26  | 7.26  | 8.55  | 22.31 | 25.00 | 27.49 | 30.58 |
| 16        | 5.81  | 6.91  | 7.96  | 9.31  | 23.54 | 26.30 | 28.85 | 32.00 |
| 17        | 6.41  | 7.56  | 8.67  | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 |
| 18        | 7.01  | 8.23  | 9.39  | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 |
| 19        | 7.63  | 8.91  | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 |
| 20        | 8.26  | 9.59  | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 |
| 21        | 8.90  | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 |
| 22        | 9.54  | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 |
| 23        | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 |
| 24        | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 |
| 25        | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 |
| 26        | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 |
| 27        | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 |
| 28        | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 |
| 29        | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 |
| 30        | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 |

**TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics  $T$ .**

| $n$ | $\alpha_{\text{two-tailed}} \leq 0.10$<br>$\alpha_{\text{one-tailed}} \leq 0.05$ | $\alpha_{\text{two-tailed}} \leq 0.05$<br>$\alpha_{\text{one-tailed}} \leq 0.025$ | $\alpha_{\text{two-tailed}} \leq 0.02$<br>$\alpha_{\text{one-tailed}} \leq 0.01$ | $\alpha_{\text{two-tailed}} \leq 0.01$<br>$\alpha_{\text{one-tailed}} \leq 0.005$ |
|-----|--|---|--|---|
| 5   | 0  |   |  |   |
| 6   | 2  | 0   |  |   |
| 7   | 3  | 2   | 0  |   |
| 8   | 5  | 3   | 1  | 0   |
| 9   | 8  | 5   | 3  | 1   |
| 10  | 10   | 8   | 5  | 3   |
| 11  | 13   | 10  | 7  | 5   |
| 12  | 17   | 13  | 9  | 7   |
| 13  | 21   | 17  | 12   | 9   |
| 14  | 25   | 21  | 15   | 12  |
| 15  | 30   | 25  | 19   | 15  |
| 16  | 35   | 29  | 23   | 19  |
| 17  | 41   | 34  | 27   | 23  |
| 18  | 47   | 40  | 32   | 27  |
| 19  | 53   | 46  | 37   | 32  |
| 20  | 60   | 52  | 43   | 37  |
| 21  | 67   | 58  | 49   | 42  |
| 22  | 75   | 65  | 55   | 48  |
| 23  | 83   | 73  | 62   | 54  |
| 24  | 91   | 81  | 69   | 61  |
| 25  | 100  | 89  | 76   | 68  |
| 26  | 110  | 98  | 84   | 75  |
| 27  | 119  | 107   | 92   | 83  |
| 28  | 130  | 116   | 101  | 91  |
| 29  | 140  | 126   | 110  | 100   |
| 30  | 151  | 137   | 120  | 109   |

Source: Adapted from McCornack, R. L. (1965). Extended tables of the Wilcoxon matched pair signed rank statistic. *Journal of the American Statistical Association*, 60, 864–871. Reprinted with permission from *The Journal of the American Statistical Association*. Copyright 1965 by the American Statistical Association. All rights reserved.

**TABLE B.4 Critical Values for the Mann–Whitney  $U$ -Test Statistic.**

| $\alpha$ | $m$ | $n$ |   |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|----------|-----|-----|---|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|--|
|          |     | 1   | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14  | 15  | 16  | 17  | 18  | 19  | 20  |  |
| 0.10     | 1   |     |   |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 2   |     |   |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 3   | 0   | 1 |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 4   | 0   | 1 | 3  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 5   | 1   | 2 | 4  | 5  |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 6   | 1   | 3 | 5  | 7  | 9  |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 7   | 1   | 4 | 6  | 8  | 11 | 13 |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 8   | 2   | 5 | 7  | 10 | 13 | 16 | 19 |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 9   | 0   | 2 | 5  | 9  | 12 | 15 | 18 | 22 | 25 |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 10  | 0   | 3 | 6  | 10 | 13 | 17 | 21 | 24 | 28 | 32 |    |    |    |     |     |     |     |     |     |     |  |
|          | 11  | 0   | 3 | 7  | 11 | 15 | 19 | 23 | 27 | 31 | 36 | 40 |    |    |     |     |     |     |     |     |     |  |
|          | 12  | 0   | 4 | 8  | 12 | 17 | 21 | 26 | 30 | 35 | 39 | 44 | 49 |    |     |     |     |     |     |     |     |  |
|          | 13  | 0   | 4 | 9  | 13 | 18 | 23 | 28 | 33 | 38 | 43 | 48 | 53 | 58 |     |     |     |     |     |     |     |  |
|          | 14  | 0   | 5 | 10 | 15 | 20 | 25 | 31 | 36 | 41 | 47 | 52 | 58 | 63 | 69  |     |     |     |     |     |     |  |
|          | 15  | 0   | 5 | 10 | 16 | 22 | 27 | 33 | 39 | 45 | 51 | 57 | 63 | 68 | 74  | 80  |     |     |     |     |     |  |
|          | 16  | 0   | 5 | 11 | 17 | 23 | 29 | 36 | 42 | 48 | 54 | 61 | 67 | 74 | 80  | 86  | 93  |     |     |     |     |  |
|          | 17  | 0   | 6 | 12 | 18 | 25 | 31 | 38 | 45 | 52 | 58 | 65 | 72 | 79 | 85  | 92  | 99  | 106 |     |     |     |  |
|          | 18  | 0   | 6 | 13 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 77 | 84 | 91  | 98  | 106 | 113 | 120 |     |     |  |
|          | 19  | 1   | 7 | 14 | 21 | 28 | 36 | 43 | 51 | 58 | 66 | 73 | 81 | 89 | 97  | 104 | 112 | 120 | 128 | 135 |     |  |
|          | 20  | 1   | 7 | 15 | 22 | 30 | 38 | 46 | 54 | 62 | 70 | 78 | 86 | 94 | 102 | 110 | 119 | 127 | 135 | 143 | 151 |  |
|          |     | 1   | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14  | 15  | 16  | 17  | 18  | 19  | 20  |  |
| 0.05     | 1   |     |   |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 2   |     |   |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 3   |     |   | 0  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 4   |     |   | 0  | 1  |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 5   | 0   | 1 | 2  | 4  |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 6   | 0   | 2 | 3  | 5  | 7  |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 7   | 0   | 2 | 4  | 6  | 8  | 11 |    |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 8   | 1   | 3 | 5  | 8  | 10 | 13 | 15 |    |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 9   | 1   | 4 | 6  | 9  | 12 | 15 | 18 | 21 |    |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 10  | 1   | 4 | 7  | 11 | 14 | 17 | 20 | 24 | 27 |    |    |    |    |     |     |     |     |     |     |     |  |
|          | 11  | 1   | 5 | 8  | 12 | 16 | 19 | 23 | 27 | 31 | 34 |    |    |    |     |     |     |     |     |     |     |  |
|          | 12  | 2   | 5 | 9  | 13 | 17 | 21 | 26 | 30 | 34 | 38 | 42 |    |    |     |     |     |     |     |     |     |  |
|          | 13  | 2   | 6 | 10 | 15 | 19 | 24 | 28 | 33 | 37 | 42 | 47 | 51 |    |     |     |     |     |     |     |     |  |
|          | 14  | 3   | 7 | 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 | 61 |     |     |     |     |     |     |     |  |
|          | 15  | 3   | 7 | 12 | 18 | 23 | 28 | 33 | 39 | 44 | 50 | 55 | 61 | 66 | 72  |     |     |     |     |     |     |  |
|          | 16  | 3   | 8 | 14 | 19 | 25 | 30 | 36 | 42 | 48 | 54 | 60 | 65 | 71 | 77  | 83  |     |     |     |     |     |  |
|          | 17  | 3   | 9 | 15 | 20 | 26 | 33 | 39 | 45 | 51 | 57 | 64 | 70 | 77 | 83  | 89  | 96  |     |     |     |     |  |
|          | 18  | 4   | 9 | 16 | 22 | 28 | 35 | 41 | 48 | 55 | 61 | 68 | 75 | 82 | 88  | 95  | 102 | 109 |     |     |     |  |
|          | 19  | 0   | 4 | 10 | 17 | 23 | 30 | 37 | 44 | 51 | 58 | 65 | 72 | 80 | 87  | 94  | 101 | 109 | 116 | 123 |     |  |
|          | 20  | 0   | 4 | 11 | 18 | 25 | 32 | 39 | 47 | 54 | 62 | 69 | 77 | 84 | 92  | 100 | 107 | 115 | 123 | 130 | 138 |  |
|          |     | 1   | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14  | 15  | 16  | 17  | 18  | 19  | 20  |  |

(Continued)

**TABLE B.4 (Continued)**

| $\alpha$ | $m$ | $n$ |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|----------|-----|-----|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
|          |     | 1   | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17  | 18  | 19  | 20  |
| 0.025    | 1   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 2   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 3   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 4   |     |   |   | 0  |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 5   |     |   | 0 | 1  | 2  |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 6   |     |   | 1 | 2  | 3  | 5  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 7   |     |   | 1 | 3  | 5  | 6  | 8  |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 8   |     | 0 | 2 | 4  | 6  | 8  | 10 | 13 |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 9   |     | 0 | 2 | 4  | 7  | 10 | 12 | 15 | 17 |    |    |    |    |    |    |    |     |     |     |     |
|          | 10  |     | 0 | 3 | 5  | 8  | 11 | 14 | 17 | 20 | 23 |    |    |    |    |    |    |     |     |     |     |
|          | 11  |     | 0 | 3 | 6  | 9  | 13 | 16 | 19 | 23 | 26 | 30 |    |    |    |    |    |     |     |     |     |
|          | 12  |     | 1 | 4 | 7  | 11 | 14 | 18 | 22 | 26 | 29 | 33 | 37 |    |    |    |    |     |     |     |     |
|          | 13  |     | 1 | 4 | 8  | 12 | 16 | 20 | 24 | 28 | 33 | 37 | 41 | 45 |    |    |    |     |     |     |     |
|          | 14  |     | 1 | 5 | 9  | 13 | 17 | 22 | 26 | 31 | 36 | 40 | 45 | 50 | 55 |    |    |     |     |     |     |
|          | 15  |     | 1 | 5 | 10 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | 54 | 59 | 64 |    |     |     |     |     |
|          | 16  |     | 1 | 6 | 11 | 15 | 21 | 26 | 31 | 37 | 42 | 47 | 53 | 59 | 64 | 70 | 75 |     |     |     |     |
|          | 17  |     | 2 | 6 | 11 | 17 | 22 | 28 | 34 | 39 | 45 | 51 | 57 | 63 | 69 | 75 | 81 | 87  |     |     |     |
|          | 18  |     | 2 | 7 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 55 | 61 | 67 | 74 | 80 | 86 | 93  | 99  |     |     |
|          | 19  |     | 2 | 7 | 13 | 19 | 25 | 32 | 38 | 45 | 52 | 58 | 65 | 72 | 78 | 85 | 92 | 99  | 106 | 113 |     |
|          | 20  |     | 2 | 8 | 14 | 20 | 27 | 34 | 41 | 48 | 55 | 62 | 69 | 76 | 83 | 90 | 98 | 105 | 112 | 119 | 127 |
| 0.01     | 1   | 1   | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17  | 18  | 19  | 20  |
|          | 2   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 3   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 4   |     |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 5   |     |   |   | 0  | 1  |    |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 6   |     |   |   | 1  | 2  | 3  |    |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 7   |     |   | 0 | 1  | 3  | 4  | 6  |    |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 8   |     |   | 0 | 2  | 4  | 6  | 7  | 9  |    |    |    |    |    |    |    |    |     |     |     |     |
|          | 9   |     |   | 1 | 3  | 5  | 7  | 9  | 11 | 14 |    |    |    |    |    |    |    |     |     |     |     |
|          | 10  |     |   | 1 | 3  | 6  | 8  | 11 | 13 | 16 | 19 |    |    |    |    |    |    |     |     |     |     |
|          | 11  |     |   | 1 | 4  | 7  | 9  | 12 | 15 | 18 | 22 | 25 |    |    |    |    |    |     |     |     |     |
|          | 12  |     |   | 2 | 5  | 8  | 11 | 14 | 17 | 21 | 24 | 28 | 31 |    |    |    |    |     |     |     |     |
|          | 13  |     | 0 | 2 | 5  | 9  | 12 | 16 | 20 | 23 | 27 | 31 | 35 | 39 |    |    |    |     |     |     |     |
|          | 14  |     | 0 | 2 | 6  | 10 | 13 | 17 | 22 | 26 | 30 | 34 | 38 | 43 | 47 |    |    |     |     |     |     |
|          | 15  |     | 0 | 3 | 7  | 11 | 15 | 19 | 24 | 28 | 33 | 37 | 42 | 47 | 51 | 56 |    |     |     |     |     |
|          | 16  |     | 0 | 3 | 7  | 12 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 | 61 | 66 |     |     |     |     |
|          | 17  |     | 0 | 4 | 8  | 13 | 18 | 23 | 28 | 33 | 38 | 44 | 49 | 55 | 60 | 66 | 71 | 77  |     |     |     |
|          | 18  |     | 0 | 4 | 9  | 14 | 19 | 24 | 30 | 36 | 41 | 47 | 53 | 59 | 65 | 70 | 76 | 82  | 88  |     |     |
|          | 19  |     | 1 | 4 | 9  | 15 | 20 | 26 | 32 | 38 | 44 | 50 | 56 | 63 | 69 | 75 | 82 | 88  | 94  | 101 |     |
|          | 20  |     | 1 | 5 | 10 | 16 | 22 | 28 | 34 | 40 | 47 | 53 | 60 | 67 | 73 | 80 | 87 | 93  | 100 | 107 | 114 |

Source: Adapted from Milton, R. C. (1964). An extended table of critical values for the Mann-Whitney (Wilcoxon) two-sample statistic. *Journal of the American Statistical Association*, 59, 925-934. Reprinted with permission from *The Journal of the American Statistical Association*. Copyright 1964 by the American Statistical Association. All rights reserved.

**TABLE B.5 Critical Values for the Friedman Test Statistic  $F_r$ .**

| $k$ | $N$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.025$ | $\alpha \leq 0.01$ |       |
|-----|-----|--------------------|--------------------|---------------------|--------------------|-------|
| 3   | 3   | 6.000              | 6.000              |                     |                    |       |
|     | 4   | 6.000              | 6.500              | 8.000               | 8.000              |       |
|     | 5   | 5.200              | 6.400              | 7.600               | 8.400              |       |
|     | 6   | 5.333              | 7.000              | 8.333               | 9.000              |       |
|     | 7   | 5.429              | 7.143              | 7.714               | 8.857              |       |
|     | 8   | 5.250              | 6.250              | 7.750               | 9.000              |       |
|     | 9   | 5.556              | 6.222              | 8.000               | 8.667              |       |
|     | 10  | 5.000              | 6.200              | 7.800               | 9.600              |       |
|     | 11  | 4.909              | 6.545              | 7.818               | 9.455              |       |
|     | 12  | 5.167              | 6.500              | 8.000               | 9.500              |       |
|     | 13  | 4.769              | 6.000              | 7.538               | 9.385              |       |
|     | 14  | 5.143              | 6.143              | 7.429               | 9.000              |       |
|     | 15  | 4.933              | 6.400              | 7.600               | 8.933              |       |
|     | 4   | 2                  | 6.000              | 6.000               |                    |       |
|     |     | 3                  | 6.600              | 7.400               | 8.200              | 9.000 |
| 4   |     | 6.300              | 7.800              | 8.400               | 9.600              |       |
| 5   |     | 6.360              | 7.800              | 8.760               | 9.960              |       |
| 6   |     | 6.400              | 7.600              | 8.800               | 10.200             |       |
| 7   |     | 6.429              | 7.800              | 9.000               | 10.371             |       |
| 8   |     | 6.300              | 7.650              | 9.000               | 10.500             |       |
| 9   |     | 6.467              | 7.800              | 9.133               | 10.867             |       |
| 10  |     | 6.360              | 7.800              | 9.120               | 10.800             |       |
| 11  |     | 6.382              | 7.909              | 9.327               | 11.073             |       |
| 12  |     | 6.400              | 7.900              | 9.200               | 11.100             |       |
| 13  |     | 6.415              | 7.985              | 7.369               | 11.123             |       |
| 14  |     | 6.343              | 7.886              | 9.343               | 11.143             |       |
| 15  |     | 6.440              | 8.040              | 9.400               | 11.240             |       |
| 5   |     | 2                  | 7.200              | 7.600               | 8.000              | 8.000 |
|     | 3   | 7.467              | 8.533              | 9.600               | 10.133             |       |
|     | 4   | 7.600              | 8.800              | 9.800               | 11.200             |       |
|     | 5   | 7.680              | 8.960              | 10.240              | 11.680             |       |
|     | 6   | 7.733              | 9.067              | 10.400              | 11.867             |       |
|     | 7   | 7.771              | 9.143              | 10.514              | 12.114             |       |
|     | 8   | 7.800              | 9.300              | 10.600              | 12.300             |       |
|     | 9   | 7.733              | 9.244              | 10.667              | 12.444             |       |
|     | 10  | 7.760              | 9.280              | 10.720              | 12.480             |       |
|     | 6   | 2                  | 8.286              | 9.143               | 9.429              | 9.714 |
| 3   |     | 8.714              | 9.857              | 10.810              | 11.762             |       |
| 4   |     | 9.000              | 10.286             | 11.429              | 12.714             |       |
| 5   |     | 9.000              | 10.486             | 11.743              | 13.229             |       |
| 6   |     | 9.048              | 10.571             | 12.000              | 13.619             |       |
| 7   |     | 9.122              | 10.674             | 12.061              | 13.857             |       |

(Continued)

**TABLE B.5 (Continued)**

| $k$ | $N$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.025$ | $\alpha \leq 0.01$ |
|-----|-----|--------------------|--------------------|---------------------|--------------------|
|     | 8   | 9.143              | 10.714             | 12.214              | 14.000             |
|     | 9   | 9.127              | 10.778             | 12.302              | 14.143             |
|     | 10  | 9.143              | 10.800             | 12.343              | 14.229             |

Source: Adapted from Martin, L., Leblanc, R., & Toan, N. K. (1993). Tables for the Friedman rank test. *The Canadian Journal of Statistics / La Revue Canadienne de Statistique*, 21(1), 39–43. Reprinted with permission from *The Canadian Journal of Statistics*. Copyright 1993 by the Statistical Society of Canada. All rights reserved.

**TABLE B.6 The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic.**

(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ ).

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 2     | 2     | 2     | 4.571429           | –                  | –                  |
| 3     | 1     | 1     | –                  | –                  | –                  |
| 3     | 2     | 1     | 4.285714           | –                  | –                  |
| 3     | 2     | 2     | 4.464286           | 4.714286           | –                  |
| 3     | 3     | 1     | 4.571429           | 5.142857           | –                  |
| 3     | 3     | 2     | 4.555556           | 5.361111           | –                  |
| 3     | 3     | 3     | 4.622222           | 5.600000           | 6.488889           |
| 4     | 2     | 1     | 4.500000           | –                  | –                  |
| 4     | 2     | 2     | 4.458333           | 5.333333           | –                  |
| 4     | 3     | 1     | 4.055556           | 5.208333           | –                  |
| 4     | 3     | 2     | 4.511111           | 5.444444           | 6.444444           |
| 4     | 3     | 3     | 4.700000           | 5.790909           | 6.745455           |
| 4     | 4     | 1     | 4.166667           | 4.966667           | 6.666667           |
| 4     | 4     | 2     | 4.554545           | 5.454545           | 7.036364           |
| 4     | 4     | 3     | 4.545455           | 5.598485           | 7.143939           |
| 4     | 4     | 4     | 4.653846           | 5.692308           | 7.653846           |
| 5     | 2     | 1     | 4.200000           | 5.000000           | –                  |
| 5     | 2     | 2     | 4.373333           | 5.160000           | 6.533333           |
| 5     | 3     | 1     | 4.017778           | 4.871111           | –                  |
| 5     | 3     | 2     | 4.650909           | 5.250909           | 6.821818           |
| 5     | 3     | 3     | 4.533333           | 5.648485           | 7.078788           |
| 5     | 4     | 1     | 3.987273           | 4.985455           | 6.954545           |
| 5     | 4     | 2     | 4.540909           | 5.272727           | 7.204545           |
| 5     | 4     | 3     | 4.548718           | 5.656410           | 7.444872           |
| 5     | 4     | 4     | 4.668132           | 5.657143           | 7.760440           |
| 5     | 5     | 1     | 4.109091           | 5.127273           | 7.309091           |
| 5     | 5     | 2     | 4.623077           | 5.338462           | 7.338462           |
| 5     | 5     | 3     | 4.545055           | 5.626374           | 7.578022           |
| 5     | 5     | 4     | 4.522857           | 5.665714           | 7.791429           |
| 5     | 5     | 5     | 4.560000           | 5.780000           | 8.000000           |

**TABLE B.6 (Continued)**(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ ).

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 6     | 2     | 1     | 4.200000           | 4.822222           | –                  |
| 6     | 2     | 2     | 4.436364           | 5.345455           | 6.654545           |
| 6     | 3     | 1     | 3.909091           | 4.854545           | 6.581818           |
| 6     | 3     | 2     | 4.681818           | 5.348485           | 6.969697           |
| 6     | 3     | 3     | 4.538462           | 5.615385           | 7.192308           |
| 6     | 4     | 1     | 4.037879           | 4.946970           | 7.083333           |
| 6     | 4     | 2     | 4.493590           | 5.262821           | 7.339744           |
| 6     | 4     | 3     | 4.604396           | 5.604396           | 7.467033           |
| 6     | 4     | 4     | 4.523810           | 5.666667           | 7.795238           |
| 6     | 5     | 1     | 4.128205           | 4.989744           | 7.182051           |
| 6     | 5     | 2     | 4.595604           | 5.318681           | 7.375824           |
| 6     | 5     | 3     | 4.535238           | 5.601905           | 7.590476           |
| 6     | 5     | 4     | 4.522500           | 5.660833           | 7.935833           |
| 6     | 5     | 5     | 4.547059           | 5.698529           | 8.027941           |
| 6     | 6     | 1     | 4.000000           | 4.857143           | 7.065934           |
| 6     | 6     | 2     | 4.438095           | 5.409524           | 7.466667           |
| 6     | 6     | 3     | 4.558333           | 5.625000           | 7.725000           |
| 6     | 6     | 4     | 4.547794           | 5.724265           | 8.000000           |
| 6     | 6     | 5     | 4.542484           | 5.764706           | 8.118954           |
| 6     | 6     | 6     | 4.538012           | 5.719298           | 8.222222           |
| 7     | 1     | 1     | 4.266667           | –                  | –                  |
| 7     | 2     | 1     | 4.200000           | 4.706494           | –                  |
| 7     | 2     | 2     | 4.525974           | 5.142857           | 7.000000           |
| 7     | 3     | 1     | 4.173160           | 4.952381           | 6.649351           |
| 7     | 3     | 2     | 4.582418           | 5.357143           | 6.838828           |
| 7     | 3     | 3     | 4.602826           | 5.620094           | 7.227630           |
| 7     | 4     | 1     | 4.120879           | 4.986264           | 6.986264           |
| 7     | 4     | 2     | 4.549451           | 5.375981           | 7.304553           |
| 7     | 4     | 3     | 4.527211           | 5.623129           | 7.498639           |
| 7     | 4     | 4     | 4.562500           | 5.650000           | 7.814286           |
| 7     | 5     | 1     | 4.035165           | 5.063736           | 7.060597           |
| 7     | 5     | 2     | 4.484898           | 5.392653           | 7.449796           |
| 7     | 5     | 3     | 4.535238           | 5.588571           | 7.697143           |
| 7     | 5     | 4     | 4.541597           | 5.732773           | 7.931092           |
| 7     | 5     | 5     | 4.540056           | 5.707563           | 8.100840           |
| 7     | 6     | 1     | 4.032653           | 5.066667           | 7.254422           |
| 7     | 6     | 2     | 4.500000           | 5.357143           | 7.490476           |
| 7     | 6     | 3     | 4.550420           | 5.672269           | 7.756303           |
| 7     | 6     | 4     | 4.561625           | 5.705882           | 8.016340           |
| 7     | 6     | 5     | 4.559733           | 5.769925           | 8.156725           |

(Continued)

**TABLE B.6 (Continued)**(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ ).

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 7     | 6     | 6     | 4.530075           | 5.730075           | 8.257143           |
| 7     | 7     | 1     | 3.985714           | 4.985714           | 7.157143           |
| 7     | 7     | 2     | 4.490546           | 5.398109           | 7.490546           |
| 7     | 7     | 3     | 4.590103           | 5.676937           | 7.809524           |
| 7     | 7     | 4     | 4.558897           | 5.765664           | 8.141604           |
| 7     | 7     | 5     | 4.545564           | 5.745564           | 8.244812           |
| 7     | 7     | 6     | 4.568027           | 5.792517           | 8.341497           |
| 7     | 7     | 7     | 4.593692           | 5.818182           | 8.378479           |
| 8     | 1     | 1     | 4.418182           | –                  | –                  |
| 8     | 2     | 1     | 4.011364           | 4.909091           | –                  |
| 8     | 2     | 2     | 4.586538           | 5.355769           | 6.663462           |
| 8     | 3     | 1     | 4.009615           | 4.881410           | 6.804487           |
| 8     | 3     | 2     | 4.450549           | 5.315934           | 6.986264           |
| 8     | 3     | 3     | 4.504762           | 5.616667           | 7.254762           |
| 8     | 4     | 1     | 4.038462           | 5.043956           | 6.972527           |
| 8     | 4     | 2     | 4.500000           | 5.392857           | 7.350000           |
| 8     | 4     | 3     | 4.529167           | 5.622917           | 7.585417           |
| 8     | 4     | 4     | 4.560662           | 5.779412           | 7.852941           |
| 8     | 5     | 1     | 3.967143           | 4.868571           | 7.110000           |
| 8     | 5     | 2     | 4.466250           | 5.415000           | 7.440000           |
| 8     | 5     | 3     | 4.514338           | 5.614338           | 7.705515           |
| 8     | 5     | 4     | 4.549020           | 5.717647           | 7.992157           |
| 8     | 5     | 5     | 4.555263           | 5.769298           | 8.115789           |
| 8     | 6     | 1     | 4.014583           | 5.014583           | 7.256250           |
| 8     | 6     | 2     | 4.441176           | 5.404412           | 7.522059           |
| 8     | 6     | 3     | 4.573529           | 5.678105           | 7.795752           |
| 8     | 6     | 4     | 4.562865           | 5.742690           | 8.045322           |
| 8     | 6     | 5     | 4.550263           | 5.750263           | 8.210263           |
| 8     | 6     | 6     | 4.598810           | 5.770238           | 8.294048           |
| 8     | 7     | 1     | 4.045431           | 5.041229           | 7.307773           |
| 8     | 7     | 2     | 4.450980           | 5.403361           | 7.571429           |
| 8     | 7     | 3     | 4.555556           | 5.698413           | 7.827068           |
| 8     | 7     | 4     | 4.548496           | 5.759211           | 8.118045           |
| 8     | 7     | 5     | 4.550612           | 5.777449           | 8.241939           |
| 8     | 7     | 6     | 4.552876           | 5.781231           | 8.332715           |
| 8     | 7     | 7     | 4.573687           | 5.795031           | 8.356296           |
| 8     | 8     | 1     | 4.044118           | 5.039216           | 7.313725           |
| 8     | 8     | 2     | 4.508772           | 5.407895           | 7.653509           |
| 8     | 8     | 3     | 4.555263           | 5.734211           | 7.889474           |
| 8     | 8     | 4     | 4.578571           | 5.742857           | 8.167857           |
| 8     | 8     | 5     | 4.572727           | 5.761039           | 8.297403           |



**TABLE B.6 (Continued)**(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ ).

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 8     | 8     | 6     | 4.572134           | 5.778656           | 8.366601           |
| 8     | 8     | 7     | 4.570652           | 5.791149           | 8.418866           |
| 8     | 8     | 8     | 4.595000           | 5.805000           | 8.465000           |
| 9     | 1     | 1     | 4.545455           | —                  | —                  |
| 9     | 2     | 1     | 3.905983           | 4.841880           | 6.346154           |
| 9     | 2     | 2     | 4.483516           | 5.260073           | 6.897436           |
| 9     | 3     | 1     | 4.073260           | 4.952381           | 6.886447           |
| 9     | 3     | 2     | 4.492063           | 5.339683           | 6.990476           |
| 9     | 3     | 3     | 4.633333           | 5.588889           | 7.355556           |
| 9     | 4     | 1     | 3.971429           | 5.071429           | 7.171429           |
| 9     | 4     | 2     | 4.488889           | 5.400000           | 7.363889           |
| 9     | 4     | 3     | 4.514706           | 5.651961           | 7.613971           |
| 9     | 4     | 4     | 4.576253           | 5.703704           | 7.909586           |
| 9     | 5     | 1     | 4.055556           | 5.040000           | 7.148889           |
| 9     | 5     | 2     | 4.464706           | 5.395588           | 7.447059           |
| 9     | 5     | 3     | 4.587364           | 5.669717           | 7.733333           |
| 9     | 5     | 4     | 4.531384           | 5.712671           | 8.024561           |
| 9     | 5     | 5     | 4.557193           | 5.769825           | 8.169825           |
| 9     | 6     | 1     | 3.933824           | 5.049020           | 7.247549           |
| 9     | 6     | 2     | 4.481481           | 5.392157           | 7.494553           |
| 9     | 6     | 3     | 4.541910           | 5.664717           | 7.822612           |
| 9     | 6     | 4     | 4.545614           | 5.744737           | 8.108772           |
| 9     | 6     | 5     | 4.573651           | 5.761905           | 8.230794           |
| 9     | 6     | 6     | 4.554113           | 5.808081           | 8.307359           |
| 9     | 7     | 1     | 4.011204           | 5.042017           | 7.270464           |
| 9     | 7     | 2     | 4.480089           | 5.429128           | 7.636591           |
| 9     | 7     | 3     | 4.535338           | 5.656140           | 7.860652           |
| 9     | 7     | 4     | 4.547732           | 5.731406           | 8.131406           |
| 9     | 7     | 5     | 4.565492           | 5.757988           | 8.287941           |
| 9     | 7     | 6     | 4.570864           | 5.782985           | 8.353284           |
| 9     | 7     | 7     | 4.583851           | 5.802622           | 8.403037           |
| 9     | 8     | 1     | 3.986355           | 4.984893           | 7.394250           |
| 9     | 8     | 2     | 4.491667           | 5.419737           | 7.642105           |
| 9     | 8     | 3     | 4.568651           | 5.717460           | 7.927381           |
| 9     | 8     | 4     | 4.559163           | 5.744228           | 8.203102           |
| 9     | 8     | 5     | 4.551252           | 5.783465           | 8.318050           |
| 9     | 8     | 6     | 4.560688           | 5.775362           | 8.408514           |
| 9     | 8     | 7     | 4.563770           | 5.807579           | 8.450000           |
| 9     | 8     | 8     | 4.582821           | 5.809744           | 8.494359           |
| 9     | 9     | 1     | 4.007018           | 4.961404           | 7.333333           |

(Continued)

**TABLE B.6 (Continued)**(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ ).

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 9     | 9     | 2     | 4.460317           | 5.411111           | 7.692063           |
| 9     | 9     | 3     | 4.565657           | 5.708514           | 7.959596           |
| 9     | 9     | 4     | 4.550066           | 5.751647           | 8.202240           |
| 9     | 9     | 5     | 4.587440           | 5.770048           | 8.370048           |
| 9     | 9     | 6     | 4.555556           | 5.814444           | 8.427778           |
| 9     | 9     | 7     | 4.567326           | 5.802198           | 8.468864           |
| 9     | 9     | 8     | 4.570750           | 5.815052           | 8.514720           |
| 9     | 9     | 9     | 4.582011           | 5.844797           | 8.564374           |
| 10    | 1     | 1     | 4.653846           | 4.653846           | –                  |
| 10    | 2     | 1     | 4.114286           | 4.839560           | 6.428571           |
| 10    | 2     | 2     | 4.434286           | 5.120000           | 6.537143           |
| 10    | 3     | 1     | 3.996190           | 5.076190           | 6.851429           |
| 10    | 3     | 2     | 4.470000           | 5.361667           | 7.041667           |
| 10    | 3     | 3     | 4.529412           | 5.588235           | 7.360294           |
| 10    | 4     | 1     | 4.042500           | 5.017500           | 7.105000           |
| 10    | 4     | 2     | 4.462500           | 5.344853           | 7.356618           |
| 10    | 4     | 3     | 4.587582           | 5.654248           | 7.616993           |
| 10    | 4     | 4     | 4.564912           | 5.715789           | 7.907018           |
| 10    | 5     | 1     | 3.988235           | 4.905882           | 7.107353           |
| 10    | 5     | 2     | 4.454902           | 5.388235           | 7.513725           |
| 10    | 5     | 3     | 4.552047           | 5.618713           | 7.752047           |
| 10    | 5     | 4     | 4.556842           | 5.744211           | 8.047895           |
| 10    | 5     | 5     | 4.574286           | 5.777143           | 8.162857           |
| 10    | 6     | 1     | 3.967320           | 5.041830           | 7.316340           |
| 10    | 6     | 2     | 4.479532           | 5.405848           | 7.588304           |
| 10    | 6     | 3     | 4.542105           | 5.655789           | 7.882105           |
| 10    | 6     | 4     | 4.550476           | 5.726190           | 8.142857           |
| 10    | 6     | 5     | 4.554978           | 5.754978           | 8.267532           |
| 10    | 6     | 6     | 4.575494           | 5.780237           | 8.338340           |
| 10    | 7     | 1     | 3.981454           | 4.985965           | 7.252130           |
| 10    | 7     | 2     | 4.491880           | 5.377444           | 7.641203           |
| 10    | 7     | 3     | 4.545034           | 5.698095           | 7.901224           |
| 10    | 7     | 4     | 4.550278           | 5.751206           | 8.172356           |
| 10    | 7     | 5     | 4.567250           | 5.763862           | 8.295652           |
| 10    | 7     | 6     | 4.563043           | 5.798758           | 8.376915           |
| 10    | 7     | 7     | 4.562286           | 5.796571           | 8.419429           |
| 10    | 8     | 1     | 3.963947           | 5.037632           | 7.358684           |
| 10    | 8     | 2     | 4.482857           | 5.429286           | 7.720714           |
| 10    | 8     | 3     | 4.533983           | 5.711688           | 7.977273           |
| 10    | 8     | 4     | 4.550988           | 5.744466           | 8.206126           |
| 10    | 9     | 5     | 4.556522           | 5.789130           | 8.344022           |

**TABLE B.6 (Continued)**

(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 3$ .)

| $n_1$ | $n_2$ | $n_3$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|--------------------|--------------------|--------------------|
| 10    | 9     | 6     | 4.573333           | 5.793833           | 8.397833           |
| 10    | 9     | 7     | 4.564484           | 5.810637           | 8.480967           |
| 10    | 9     | 8     | 4.561538           | 5.829060           | 8.494017           |
| 10    | 9     | 1     | 4.025714           | 4.988571           | 7.436508           |
| 10    | 9     | 2     | 4.476479           | 5.446176           | 7.693795           |
| 10    | 9     | 3     | 4.570751           | 5.700659           | 7.997628           |
| 10    | 9     | 4     | 4.556401           | 5.757609           | 8.223430           |
| 10    | 9     | 5     | 4.547556           | 5.792000           | 8.380222           |
| 10    | 9     | 6     | 4.561231           | 5.813128           | 8.449436           |
| 10    | 9     | 7     | 4.559707           | 5.817610           | 8.507475           |
| 10    | 9     | 8     | 4.567063           | 5.833730           | 8.544489           |
| 10    | 9     | 9     | 4.578982           | 5.830706           | 8.575698           |
| 10    | 10    | 1     | 3.987013           | 5.054545           | 7.501299           |
| 10    | 10    | 2     | 4.477470           | 5.449802           | 7.726482           |
| 10    | 10    | 3     | 4.559420           | 5.687681           | 8.026087           |
| 10    | 10    | 4     | 4.567000           | 5.776000           | 8.263000           |
| 10    | 10    | 5     | 4.554462           | 5.793231           | 8.403692           |
| 10    | 10    | 6     | 4.561823           | 5.796011           | 8.472934           |
| 10    | 10    | 7     | 4.558277           | 5.820408           | 8.536508           |
| 10    | 10    | 8     | 4.565025           | 5.837438           | 8.565887           |
| 10    | 10    | 9     | 4.567050           | 5.837241           | 8.606130           |
| 10    | 10    | 10    | 4.583226           | 5.855484           | 8.640000           |

(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 4$ .)

| $n_1$ | $n_2$ | $n_3$ | $n_4$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|-------|--------------------|--------------------|--------------------|
| 2     | 2     | 2     | 1     | 5.357143           | 5.678571           | –                  |
| 2     | 2     | 2     | 2     | 5.666667           | 6.166667           | 6.666667           |
| 3     | 2     | 1     | 1     | 4.892857           | –                  | –                  |
| 3     | 2     | 2     | 1     | 5.555556           | 5.833333           | –                  |
| 3     | 2     | 2     | 2     | 5.644444           | 6.333333           | 7.133333           |
| 3     | 3     | 1     | 1     | 5.333333           | 6.333333           | –                  |
| 3     | 3     | 2     | 1     | 5.622222           | 6.244444           | 7.044444           |
| 3     | 3     | 2     | 2     | 5.745455           | 6.527273           | 7.636364           |
| 3     | 3     | 3     | 1     | 5.654545           | 6.600000           | 7.400000           |
| 3     | 3     | 3     | 2     | 5.878788           | 6.727273           | 8.015152           |
| 3     | 3     | 3     | 3     | 5.974359           | 6.897436           | 8.435897           |
| 4     | 2     | 1     | 1     | 5.250000           | 5.833333           | –                  |
| 4     | 2     | 2     | 1     | 5.533333           | 6.133333           | 7.000000           |
| 4     | 2     | 2     | 2     | 5.754545           | 6.545455           | 7.390909           |
| 4     | 3     | 1     | 1     | 5.066667           | 6.177778           | 7.066667           |

(Continued)

**TABLE B.6 (Continued)**(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 4$ .)

| $n_1$ | $n_2$ | $n_3$ | $n_4$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|-------|--------------------|--------------------|--------------------|
| 4     | 3     | 2     | 1     | 5.572727           | 6.309091           | 7.454545           |
| 4     | 3     | 2     | 2     | 5.750000           | 6.621212           | 7.871212           |
| 4     | 3     | 3     | 1     | 5.666667           | 6.545455           | 7.757576           |
| 4     | 3     | 3     | 2     | 5.858974           | 6.782051           | 8.320513           |
| 4     | 3     | 3     | 3     | 6.000000           | 6.967033           | 8.653846           |
| 4     | 4     | 1     | 1     | 5.181818           | 5.945455           | 7.909091           |
| 4     | 4     | 2     | 1     | 5.568182           | 6.386364           | 7.909091           |
| 4     | 4     | 2     | 2     | 5.807692           | 6.730769           | 8.346154           |
| 4     | 4     | 3     | 1     | 5.660256           | 6.634615           | 8.217949           |
| 4     | 4     | 3     | 2     | 5.901099           | 6.873626           | 8.620879           |
| 4     | 4     | 3     | 3     | 6.004762           | 7.038095           | 8.866667           |
| 4     | 4     | 4     | 1     | 5.653846           | 6.725275           | 8.587912           |
| 4     | 4     | 4     | 2     | 5.914286           | 6.957143           | 8.871429           |
| 4     | 4     | 4     | 3     | 6.029167           | 7.129167           | 9.075000           |
| 4     | 4     | 4     | 4     | 6.088235           | 7.235294           | 9.286765           |
| 5     | 1     | 1     | 1     | 5.333333           | —                  | —                  |
| 5     | 2     | 1     | 1     | 5.266667           | 5.960000           | —                  |
| 5     | 2     | 2     | 1     | 5.541818           | 6.109091           | 7.276364           |
| 5     | 2     | 2     | 2     | 5.636364           | 6.563636           | 7.772727           |
| 5     | 3     | 1     | 1     | 5.130909           | 6.003636           | 7.400000           |
| 5     | 3     | 2     | 1     | 5.518182           | 6.363636           | 7.757576           |
| 5     | 3     | 2     | 2     | 5.771795           | 6.664103           | 8.202564           |
| 5     | 3     | 3     | 1     | 5.656410           | 6.641026           | 8.117949           |
| 5     | 3     | 3     | 2     | 5.865934           | 6.821978           | 8.606593           |
| 5     | 3     | 3     | 3     | 6.020952           | 7.011429           | 8.840000           |
| 5     | 4     | 1     | 1     | 5.254545           | 6.040909           | 7.909091           |
| 5     | 4     | 2     | 1     | 5.580769           | 6.419231           | 8.173077           |
| 5     | 4     | 2     | 2     | 5.782418           | 6.725275           | 8.472527           |
| 5     | 4     | 3     | 1     | 5.639560           | 6.681319           | 8.408791           |
| 5     | 4     | 3     | 2     | 5.901905           | 6.925714           | 8.801905           |
| 5     | 4     | 3     | 3     | 6.029167           | 7.093333           | 9.029167           |
| 5     | 4     | 4     | 1     | 5.674286           | 6.760000           | 8.725714           |
| 5     | 4     | 4     | 2     | 5.947500           | 6.990000           | 9.002500           |
| 5     | 4     | 4     | 3     | 6.035294           | 7.172794           | 9.220588           |
| 5     | 4     | 4     | 4     | 6.066667           | 7.262745           | 9.392157           |
| 5     | 5     | 1     | 1     | 5.153846           | 6.076923           | 8.107692           |
| 5     | 5     | 2     | 1     | 5.564835           | 6.540659           | 8.327473           |
| 5     | 5     | 2     | 2     | 5.794286           | 6.777143           | 8.634286           |
| 5     | 5     | 3     | 1     | 5.662857           | 6.737143           | 8.611429           |
| 5     | 5     | 3     | 2     | 5.921667           | 6.946667           | 8.946667           |
| 5     | 5     | 3     | 3     | 6.023529           | 7.117647           | 9.188235           |

**TABLE B.6 (Continued)**

(The Critical Values for the Kruskal–Wallis  $H$ -Test Statistic,  $k = 4$ .)

| $n_1$ | $n_2$ | $n_3$ | $n_4$ | $\alpha \leq 0.10$ | $\alpha \leq 0.05$ | $\alpha \leq 0.01$ |
|-------|-------|-------|-------|--------------------|--------------------|--------------------|
| 5     | 5     | 4     | 1     | 5.670000           | 6.782500           | 8.870000           |
| 5     | 5     | 4     | 2     | 5.944853           | 7.032353           | 9.156618           |
| 5     | 5     | 4     | 3     | 6.052288           | 7.217647           | 9.356863           |
| 5     | 5     | 4     | 4     | 6.070175           | 7.291228           | 9.536842           |
| 5     | 5     | 5     | 1     | 5.682353           | 6.829412           | 9.052941           |
| 5     | 5     | 5     | 2     | 5.945098           | 7.074510           | 9.286275           |
| 5     | 5     | 5     | 3     | 6.043275           | 7.250292           | 9.495906           |
| 5     | 5     | 5     | 4     | 6.082105           | 7.327895           | 9.669474           |
| 5     | 5     | 5     | 5     | 6.097143           | 7.377143           | 9.800000           |

Source: Adapted from Meyer, J. P., & Seaman, M. A. (2008, March). A comparison of the exact Kruskal-Wallis distribution to asymptotic approximations for  $N \leq 105$ . Paper presented at the annual meeting of the American Educational Research Association, New York. Reprinted with permission of the authors.

**TABLE B.7 Critical Values for the Spearman Rank-Order Correlation Coefficient  $r_s$ .**

| $n$ | $\alpha_{\text{two-tailed}} \leq 0.10$<br>$\alpha_{\text{one-tailed}} \leq 0.05$ | $\alpha_{\text{two-tailed}} \leq 0.05$<br>$\alpha_{\text{one-tailed}} \leq 0.025$ | $\alpha_{\text{two-tailed}} \leq 0.02$<br>$\alpha_{\text{one-tailed}} \leq 0.01$ | $\alpha_{\text{two-tailed}} \leq 0.01$<br>$\alpha_{\text{one-tailed}} \leq 0.005$ |
|-----|--|---|--|---|
| 4   | 1.000  |   |  |   |
| 5   | 0.900  | 1.000   | 1.000  |   |
| 6   | 0.829  | 0.886   | 0.943  | 1.000   |
| 7   | 0.714  | 0.786   | 0.893  | 0.929   |
| 8   | 0.643  | 0.738   | 0.833  | 0.881   |
| 9   | 0.600  | 0.700   | 0.783  | 0.833   |
| 10  | 0.564  | 0.648   | 0.745  | 0.794   |
| 11  | 0.536  | 0.618   | 0.709  | 0.755   |
| 12  | 0.503  | 0.587   | 0.671  | 0.727   |
| 13  | 0.484  | 0.560   | 0.648  | 0.703   |
| 14  | 0.464  | 0.538   | 0.622  | 0.675   |
| 15  | 0.443  | 0.521   | 0.604  | 0.654   |
| 16  | 0.429  | 0.503   | 0.582  | 0.635   |
| 17  | 0.414  | 0.485   | 0.566  | 0.615   |
| 18  | 0.401  | 0.472   | 0.550  | 0.600   |
| 19  | 0.391  | 0.460   | 0.535  | 0.584   |
| 20  | 0.380  | 0.447   | 0.520  | 0.570   |
| 21  | 0.370  | 0.435   | 0.508  | 0.556   |
| 22  | 0.361  | 0.425   | 0.496  | 0.544   |
| 23  | 0.353  | 0.415   | 0.486  | 0.532   |

(Continued)

**TABLE B.7 (Continued)**

| $n$ | $\alpha_{\text{two-tailed}} \leq 0.10$<br>$\alpha_{\text{one-tailed}} \leq 0.05$ | $\alpha_{\text{two-tailed}} \leq 0.05$<br>$\alpha_{\text{one-tailed}} \leq 0.025$ | $\alpha_{\text{two-tailed}} \leq 0.02$<br>$\alpha_{\text{one-tailed}} \leq 0.01$ | $\alpha_{\text{two-tailed}} \leq 0.01$<br>$\alpha_{\text{one-tailed}} \leq 0.005$ |
|-----|--|---|--|---|
| 24  | 0.344  | 0.406   | 0.476  | 0.321   |
| 25  | 0.337  | 0.398   | 0.466  | 0.511   |
| 26  | 0.331  | 0.390   | 0.457  | 0.501   |
| 27  | 0.324  | 0.382   | 0.448  | 0.491   |
| 28  | 0.317  | 0.375   | 0.440  | 0.483   |
| 29  | 0.312  | 0.368   | 0.433  | 0.475   |
| 30  | 0.306  | 0.362   | 0.425  | 0.467   |
| 31  | 0.301  | 0.356   | 0.418  | 0.459   |
| 32  | 0.296  | 0.350   | 0.412  | 0.452   |
| 33  | 0.291  | 0.345   | 0.405  | 0.446   |
| 34  | 0.287  | 0.340   | 0.399  | 0.439   |
| 35  | 0.283  | 0.335   | 0.394  | 0.433   |
| 36  | 0.279  | 0.330   | 0.388  | 0.427   |
| 37  | 0.275  | 0.325   | 0.383  | 0.421   |
| 38  | 0.271  | 0.321   | 0.378  | 0.415   |
| 39  | 0.267  | 0.317   | 0.373  | 0.410   |
| 40  | 0.264  | 0.313   | 0.368  | 0.405   |
| 41  | 0.261  | 0.309   | 0.364  | 0.400   |
| 42  | 0.257  | 0.305   | 0.359  | 0.395   |
| 43  | 0.254  | 0.301   | 0.355  | 0.391   |
| 44  | 0.251  | 0.298   | 0.351  | 0.386   |
| 45  | 0.248  | 0.294   | 0.347  | 0.382   |
| 46  | 0.246  | 0.291   | 0.343  | 0.378   |
| 47  | 0.243  | 0.288   | 0.340  | 0.374   |
| 48  | 0.240  | 0.285   | 0.336  | 0.370   |
| 49  | 0.238  | 0.282   | 0.333  | 0.366   |
| 50  | 0.235  | 0.279   | 0.329  | 0.363   |

Source: Adapted from Zar, J. H. (1972). Significance testing of the Spearman rank correlation coefficient. *Journal of the American Statistical Association*, 67, 578–580. Reprinted with permission from The *Journal of the American Statistical Association*. Copyright 1972 by the American Statistical Association. All rights reserved.

**TABLE B.8 Critical Values for the Pearson Product-Moment Correlation Coefficient  $r$ .**

| $df$ | $\alpha$ two-tailed $\leq 0.10$<br>$\alpha$ one-tailed $\leq 0.05$ | $\alpha$ two-tailed $\leq 0.05$<br>$\alpha$ one-tailed $\leq 0.025$ | $\alpha$ two-tailed $\leq 0.025$<br>$\alpha$ one-tailed $\leq 0.0125$ | $\alpha$ two-tailed $\leq 0.01$<br>$\alpha$ one-tailed $\leq 0.005$ |
|------|--|---|---|---|
| 1    | 0.988  | 0.997   | 0.999   | 0.999   |
| 2    | 0.900  | 0.950   | 0.975   | 0.990   |
| 3    | 0.805  | 0.878   | 0.924   | 0.959   |
| 4    | 0.729  | 0.811   | 0.868   | 0.917   |
| 5    | 0.669  | 0.754   | 0.817   | 0.875   |
| 6    | 0.621  | 0.707   | 0.771   | 0.834   |
| 7    | 0.582  | 0.666   | 0.732   | 0.798   |
| 8    | 0.549  | 0.632   | 0.697   | 0.765   |
| 9    | 0.521  | 0.602   | 0.667   | 0.735   |
| 10   | 0.497  | 0.576   | 0.640   | 0.708   |
| 11   | 0.476  | 0.553   | 0.616   | 0.684   |
| 12   | 0.458  | 0.532   | 0.594   | 0.661   |
| 13   | 0.441  | 0.514   | 0.575   | 0.641   |
| 14   | 0.426  | 0.497   | 0.557   | 0.623   |
| 15   | 0.412  | 0.482   | 0.541   | 0.606   |
| 16   | 0.400  | 0.468   | 0.526   | 0.590   |
| 17   | 0.389  | 0.456   | 0.512   | 0.575   |
| 18   | 0.378  | 0.444   | 0.499   | 0.561   |
| 19   | 0.369  | 0.433   | 0.487   | 0.549   |
| 20   | 0.360  | 0.423   | 0.476   | 0.537   |
| 21   | 0.352  | 0.413   | 0.466   | 0.526   |
| 22   | 0.344  | 0.404   | 0.456   | 0.515   |
| 23   | 0.337  | 0.396   | 0.447   | 0.505   |
| 24   | 0.330  | 0.388   | 0.439   | 0.496   |
| 25   | 0.323  | 0.381   | 0.430   | 0.487   |
| 26   | 0.317  | 0.374   | 0.423   | 0.479   |
| 27   | 0.311  | 0.367   | 0.415   | 0.471   |
| 28   | 0.306  | 0.361   | 0.409   | 0.463   |
| 29   | 0.301  | 0.355   | 0.402   | 0.456   |
| 30   | 0.296  | 0.349   | 0.396   | 0.449   |
| 31   | 0.291  | 0.344   | 0.390   | 0.442   |
| 32   | 0.287  | 0.339   | 0.384   | 0.436   |
| 33   | 0.283  | 0.334   | 0.378   | 0.430   |
| 34   | 0.279  | 0.329   | 0.373   | 0.424   |
| 35   | 0.275  | 0.325   | 0.368   | 0.418   |
| 36   | 0.271  | 0.320   | 0.363   | 0.413   |
| 37   | 0.267  | 0.316   | 0.359   | 0.408   |
| 38   | 0.264  | 0.312   | 0.354   | 0.403   |
| 39   | 0.260  | 0.308   | 0.350   | 0.398   |
| 40   | 0.257  | 0.304   | 0.346   | 0.393   |
| 41   | 0.254  | 0.301   | 0.342   | 0.389   |

(Continued)

**TABLE B.8 (Continued)**

| <i>df</i> | $\alpha_{\text{two-tailed}} \leq 0.10$ | $\alpha_{\text{two-tailed}} \leq 0.05$  | $\alpha_{\text{two-tailed}} \leq 0.025$  | $\alpha_{\text{two-tailed}} \leq 0.01$  |
|-----------|--|---|--|---|
|           | $\alpha_{\text{one-tailed}} \leq 0.05$ | $\alpha_{\text{one-tailed}} \leq 0.025$ | $\alpha_{\text{one-tailed}} \leq 0.0125$ | $\alpha_{\text{one-tailed}} \leq 0.005$ |
| 42        | 0.251                                  | 0.297                                   | 0.338                                    | 0.384                                   |
| 43        | 0.248                                  | 0.294                                   | 0.334                                    | 0.380                                   |
| 44        | 0.246                                  | 0.291                                   | 0.330                                    | 0.376                                   |
| 45        | 0.243                                  | 0.288                                   | 0.327                                    | 0.372                                   |
| 46        | 0.240                                  | 0.285                                   | 0.323                                    | 0.368                                   |
| 47        | 0.238                                  | 0.282                                   | 0.320                                    | 0.365                                   |
| 48        | 0.235                                  | 0.279                                   | 0.317                                    | 0.361                                   |
| 49        | 0.233                                  | 0.276                                   | 0.314                                    | 0.358                                   |
| 50        | 0.231                                  | 0.273                                   | 0.311                                    | 0.354                                   |

**TABLE B.9 Factorials.**

| <i>n</i> | <i>n!</i>                          |
|----------|------------------------------------|
| 1        | 1                                  |
| 2        | 2                                  |
| 3        | 6                                  |
| 4        | 24                                 |
| 5        | 120                                |
| 6        | 720                                |
| 7        | 5040                               |
| 8        | 40,320                             |
| 9        | 362,880                            |
| 10       | 3,628,800                          |
| 11       | 39,916,800                         |
| 12       | 479,001,600                        |
| 13       | 6,227,020,800                      |
| 14       | 87,178,291,200                     |
| 15       | 1,307,674,368,000                  |
| 16       | 20,922,789,888,000                 |
| 17       | 355,687,428,096,000                |
| 18       | 6,402,373,705,728,000              |
| 19       | 121,645,100,408,832,000            |
| 20       | 2,432,902,008,176,640,000          |
| 21       | 51,090,942,171,709,440,000         |
| 22       | 1,124,000,727,777,607,680,000      |
| 23       | 25,852,016,738,884,976,640,000     |
| 24       | 620,448,401,733,239,439,360,000    |
| 25       | 15,511,210,043,330,985,984,000,000 |



**TABLE B.10 Critical Values for the Runs Test for Randomness.**

One-tailed alternative;  $\alpha = 0.05$ .

| $n_1$ | $n_2$ |   |   |   |   |   |   |   |    |    |    |
|-------|-------|---|---|---|---|---|---|---|----|----|----|
|       | 2     | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2     | -     | - | - | - | - | - | 2 | 2 | 2  | 2  | 2  |
| 3     | -     | - | - | 2 | 2 | 2 | 2 | 2 | 3  | 3  | 3  |
| 4     | -     | - | 2 | 2 | 3 | 3 | 3 | 3 | 3  | 3  | 4  |
| 5     | -     | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4  | 4  | 4  |
| 6     | -     | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5  | 5  | 5  |
| 7     | -     | 2 | 3 | 3 | 4 | 4 | 4 | 5 | 5  | 5  | 6  |
| 8     | 2     | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6  | 6  | 6  |
| 9     | 2     | 2 | 3 | 4 | 4 | 5 | 5 | 6 | 6  | 6  | 7  |
| 10    | 2     | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 6  | 7  | 7  |
| 11    | 2     | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7  | 7  | 8  |
| 12    | 2     | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 7  | 8  | 8  |

One-tailed alternative;  $\alpha = 0.025$ .

| $n_1$ | $n_2$ |   |   |   |   |   |   |   |    |    |    |
|-------|-------|---|---|---|---|---|---|---|----|----|----|
|       | 2     | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2     | -     | - | - | - | - | - | - | - | -  | -  | 2  |
| 3     | -     | - | - | - | 2 | 2 | 2 | 2 | 2  | 2  | 2  |
| 4     | -     | - | - | 2 | 2 | 2 | 3 | 3 | 3  | 3  | 3  |
| 5     | -     | - | 2 | 2 | 3 | 3 | 3 | 3 | 3  | 4  | 4  |
| 6     | -     | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4  | 4  | 4  |
| 7     | -     | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 5  | 5  | 5  |

(Continued)

**TABLE B.10 (Continued)**One-tailed alternative;  $\sigma = 0.025$ .

| $n_1$ | $n_2$ |   |   |    |    |    |    |    |    |    |    |
|-------|-------|---|---|----|----|----|----|----|----|----|----|
|       | 2     | 3 | 4 | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
| 8     | –     | 2 | 3 | 3  | 3  | 4  | 4  | 5  | 5  | 5  | 6  |
|       | –     | – | – | 11 | 12 | 13 | 14 | 14 | 15 | 15 | 16 |
| 9     | –     | 2 | 3 | 3  | 4  | 4  | 5  | 5  | 5  | 6  | 6  |
|       | –     | – | – | –  | 13 | 14 | 14 | 15 | 16 | 16 | 16 |
| 10    | –     | 2 | 3 | 3  | 4  | 5  | 5  | 5  | 6  | 6  | 7  |
|       | –     | – | – | –  | 13 | 14 | 15 | 16 | 16 | 17 | 17 |
| 11    | –     | 2 | 3 | 4  | 4  | 5  | 5  | 6  | 6  | 7  | 7  |
|       | –     | – | – | –  | 13 | 14 | 15 | 16 | 17 | 17 | 18 |
| 12    | 2     | 2 | 3 | 4  | 4  | 5  | 6  | 6  | 7  | 7  | 7  |
|       | –     | – | – | –  | 13 | 14 | 16 | 16 | 17 | 18 | 19 |

Source: Adapted from tables D.5 and D.6 of Janke, S. J., & Tinsley, F. C. (2005). *Introduction to Linear Models and Statistical Inference*. Hoboken, NJ: John Wiley & Sons, Inc. Reprinted with permission of John Wiley & Sons, Inc. Copyright 2005 by John Wiley & Sons, Inc. All rights reserved.