King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2016–2017
Academic Year (H) 1437–1438
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Test Sunday April 16, 2017, from 7 to 9 PM

Exercise 1.

We specify below the basic elements of a financial market with T periods:

- A finite probability space $\Omega = \{\omega_1, \dots, \omega_k\}$ with k elements.
- A probability measure P on Ω, such that $P(\omega) > 0$ for all $\omega \in \Omega$.
- A riskless asset (a saving account) $S_t^0, t \in \{0, 1, 2, \dots T\}$ such that $S_0^0 = 1$ with a constant interest rate r.
- A d–dimensional price process S_t, $t \in \{0, 1, 2, \dots T\}$ where $S_t = (S_t^0, S_t^1, \dots, S_t^d)$ and S_t^i stands for the price of the asset i at time t.

1. **(1 mark)** Give the definition of a portfolio in this market
2. **(1 mark)** Recall the self–financing property for this model
3. **(1 mark)** Give the definition of attainable payoffs for this model
4. **(1 mark)** Give the definition of a RNPM (risk neutral probability measure) in this setting.
5. **(1 mark)** Give the definition of a complete market
6. **(1 mark)** Give the definition of an incomplete market

Exercise 2.

Consider stock with a current price \$100 and a constant annualized volatility σ of 20%. The stock does not pay dividends. A risk–less asset is worth \$0.95 today and is worth \$1 in one year maturity.
Consider also European and American put options on the stock with a maturity of **two years** and a strike price of \$110.

1. Build a **two–step** binomial tree of the stock price, with each step being **one year**.
2. Find the risk–neutral probability measure if any.

3. Is the model arbitrage free ? and complete.
4. Give the binomial tree of the European put option.
5. Is this European put option attainable in this market?
6. If yes find its replicating portfolio of the European put option.
7. Is an American put option attainable in this market?
8. If yes find its replicating portfolio of the American put option.

Exercise 3.

Consider the following model

n	S_n^0	S_n^1			S_n^2		
		ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0	1	5	5	5	10	10	10
1	$\frac{10}{9}$	$\frac{60}{9}$	$\frac{60}{9}$	$\frac{30}{9}$	$\frac{120}{9}$	$\frac{80}{9}$	$\frac{80}{9}$

1. Is the market $(S_t^0, S_t^1)_{t\in\{0,1\}}$ arbitrage free ?
2. Is the market $(S_t^0, S_t^1)_{t\in\{0,1\}}$ complete ?
3. Give an example of a contingent claim. Is it attainable ?
4. Find a replicating portfolio for your contingent claim.

Exercise 4.

1. Calculate u, d, and the RNPM q when a binomial tree is constructed to value an option on a foreign currency. The tree step size is 2 month, the domestic interest rate is 6% per annum, the foreign interest rate is 4% per annum, and the volatility is 12% per annum.
2. **(1 mark)** Build the two–month step binomial algorithm for a European call option on a foreign currency with initial price $S_0 = 1.50$, maturity six–months and strike price $K = 1.45$ using the parameters given in Q1.
3. **(1 mark)** Use the two–month step binomial tree to find the initial price (premium) of an American call option on a foreign currency with initial price 1.50, maturity six–months and strike price 1.45 using the parameters given in Q1.
4. Compare the two prices obtained in Q2 and Q3.