
FUNCTIONS OF RANDOM VARIABLES

i. **METHOD OF DISTRIBUTION FUNCTIONS**

ii. **ONE-TO-ONE TRANSFORMATIONS.**

iii. **TWO-TO TWO TRANSFORMATIONS. (JOINT DISTRIBUTION OF FUNCTIONS OF RANDOM VARIABLES)**

iv. **METHOD OF MOMENT-GENERATING FUNCTIONS.**

Q1) If $X \sim Uniform(0,1)$, find the pdf of $Y = -2lnX$. Name the distribution and its parameter values.

Q2) If $X \sim Normal(\mu, \sigma^2)$, find the pdf of $Y = e^X$.

Q3) If $X \sim Exponential(1)$, find the pdf of $Y = -lnX$.

Q4) If $X \sim Uniform(0,1)$, find the pdf of $Y = \sqrt{X}$.

Q5) The pdf of X is given by $f_X(x) = \frac{1}{2}x\ ; 0 < x < 2$.

a. Find the pdf of $Y = X^3$.

b. Find $P\left(\frac{1}{2} < X < 1\right)$ and $P\left(\frac{1}{8} < Y < 1\right)$. Are they the same or different? Why?

Q6) If $X \sim Uniform(0,1)$ independent of $Y \sim Exponential(1)$, find the distribution of $Z = X + Y$.

Q7) Let X_1 and X_2 be independent $Exp(\lambda)$ r.v. Find the joint density of $Y_1 = X_1 + X_2$ and $Y_2 = e^{X_1}$.

Q8) Let $X_1 \sim Exp(\lambda_1)$ independent of $X_2 \sim Exp(\lambda_2)$ r.v.. Find:

a. The cumulative distribution function of $Z = \frac{X_1}{X_2}$.

b. $P(X_1 < X_2)$.

Q9) Let X and Y be distributed as independent $Uniform(0,1)$ r.v. ,

a. Find the joint density function of $Z_1 = X + Y$ and $Z_2 = Y$.

b. Find the marginal pdf of Z_1 from the joint density.

Q10) Let X and Y be distributed as independent $Exp(1)$ r.v., find:

a. The joint density function of $Z = X + Y$ and $U = \frac{X}{X+Y}$.

b. Find the marginal pdf of U.

Q11) Let X and Y have independent $Gamma(\alpha, \lambda)$ distributions.

a. Find the joint pdf of $U = \frac{X}{X+Y}$ and $V = X + Y$.

b. Show that the marginal density of U is a Beta distribution.

- If X_i $indpt.$ $Exp(\lambda), then\ the\ sum\ \sum_{i=1}^{i=n} X_i\ \sim Gamma\ (n, \lambda)$
- If X_i $indpt.$ $Gamma(\alpha_i, \beta), then\ the\ sum\ \sum_{i=1}^{i=n} X_i\ \sim Gamma\ (\sum_{i=1}^{i=n} \alpha_i\ , \beta)$
- If X_i $indpt.$ $Normal(\mu_i, \sigma_i^2), then\ the\ sum\ \sum_{i=1}^{i=n} X_i\ \sim Normal\ (\sum_{i=1}^{i=n} \mu_i\ , \sum_{i=1}^{i=n} \sigma_i^2)$
- If X_i $indpt.$ $Normal(\mu_0, \sigma_0^2), then\ the\ sum\ \sum_{i=1}^{i=n} X_i\ \sim Normal\ (n\mu_0, n\sigma_0^2)$
- If $Z\ \ Normal(0,1), then\ the\ \ Z^2 \sim \chi_{\mathbf{1}}^{\mathbf{2}}$
- If $X\ \sim \chi_{\mathbf{n}}^{\mathbf{2}},\ \ ind, of\ Y \sim \chi_{\mathbf{m},}^{\mathbf{2}}\ \ then\ the\ \ X + Y \sim \chi_{\mathbf{n+m}}^{\mathbf{2}}$
- If $Z_1\ \ Normal(0,1), ind. of\ Z_2\ \ Normal(0,1)\ then\ the\ Z_1 + Z_2 \sim \chi_{\mathbf{2}}^{\mathbf{2}}$