
RANDOM VARIABLES, DISTRIBUTIONS AND EXPECTATIONS

DISCRETE DISTRIBUTIONS:

Q1. Consider the experiment of flipping a balanced coin three times independently.
Let X= Number of heads – Number of tails.

(a) List the elements of the sample space S.
(b) Assign a value x of X to each sample point.
(c) Find the probability distribution function of X.
(d) Find $P(X \leq 1)$
(e) Find $P(X < 1)$
(f) Find $\mu = E(X)$
(g) Find $\sigma^2 = Var(X)$

Q2. Let X be a random variable with the following probability distribution:

x	–3	6	9
f(x)	0.1	0.5	0.4

1) Find the mean (expected value) of X, $\mu = E(X)$.
2) Find $E(X^2)$.
3) Find the variance of X, Var (X) = σ_X^2 .
4) Find the mean of 2X+1, $E(2X+1) = \mu_{2X+1}$.
5) Find the variance of 2X+1, Var(2X+1)= σ_{2X+1}^2.

Q3. Which of the following is a probability distribution function:

(A) $f(x) = \frac{x+1}{10}$; x=0,1,2,3,4 (B) $f(x) = \frac{x-1}{5}$; x=0,1,2,3,4

(C) $f(x) = \frac{1}{5}$; x=0,1,2,3,4 (D) $f(x) = \frac{5-x^2}{6}$; x=0,1,2,3

Q4. Let X be a discrete random variable with the probability distribution function:
$f(x) = kx$ for x=1, 2, and 3.

(i) Find the value of *k*.
(ii) Find the cumulative distribution function (CDF), $F(x)$.
(iii) Using the CDF, $F(x)$, find $P(0.5 < X \leq 2.5)$.

Q5. Let X be a random variable with cumulative distribution function (CDF) given by:

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25, & 0 \leq x < 1 \\ 0.6, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

(a) Find the probability distribution function of X, f(x).
(b) Find $P(1 \leq X < 2)$. (using both f(x) and F(x))
(c) Find $P(X > 2)$. (using both f(x) and F(x))

Q6. Consider the random variable X with the following probability distribution function:

x	0	1	2	3
$f(x)$	0.4	c	0.3	0.1

The value of c is

(A) 0.125 (B) 0.2 (C) 0.1 (D) 0.125 (E) -0.2

Q7. The probability distribution for company A is given by:

x	1	2	3
f(x)	0.3	0.4	0.3

and for company B is given by:

y	0	1	2	3	4
f(y)	0.2	0.1	0.3	0.3	0.1

Show that the variance of the probability distribution for company B is greater than that of company A.

<u>CONTINUOUS DISTRIBUTIONS:</u>

Q1. If the continuous random variable X has mean μ=16 and variance σ^2=5, then $P(X = 16)$ is

(A) 0.0625 (B) 0.5 (C) 0.0 (D) None of these.

Q2. Consider the probability density function:

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

1) The value of k is:

(A) 1 (B) 0.5 (C) 1.5 (D) 0.667

2) The probability $P(0.3 < X \le 0.6)$ is,

(A) 0.4647 (B) 0.3004 (C) 0.1643 (D) 0.4500

3) The expected value of X, $E(X)$ is,

(A) 0.6 (B) 1.5 (C) 1 (D) 0.667

[Hint: $\int \sqrt{x}\, dx = \frac{x^{3/2}}{(3/2)} + c$]

Q3. If the cumulative distribution function of the random variable X having the form:

$$P(X \le x) = F(x) = \begin{cases} 0 & ; x < 0 \\ x/(x+1) & ; x \ge 0 \end{cases}$$

Then

(1) $P(0 < X < 2)$ equals to

(a) 0.555 (b) 0.333 (c) 0.667 (d) none of these.

(2) If $P(X \le k) = 0.5$, then k equals to

(a) 5 (b) 0.5 (c) 1 (d) 1.5

Q4) For each function below, determine if it can be probability density function. If so, determine c.

a. $f_1(x) = c(2x - x^3)\ ; for\ 0 < x < \frac{5}{2}$
b. $f_2(x) = c(2x - x^2)\ ; for\ 0 < x < \frac{5}{2}$
c. $f_3(x) = c(2x^2 - 4x)\ ; for\ 0 < x < 3$
d. $f_4(x) = c(2x^2 - 4x)\ ; for\ 0 < x < 2$

Q5) The r.v. X has pdf $f(x) = \begin{cases} c(1 - x^2) & ; for - 1 < x < 1 \\ 0 & ; otherwise \end{cases}$

a. What is the value of c.
b. Find the following probabilities using the pdf of X:
  i. $P(X < 0)$
  ii. $P\left(X \geq \frac{1}{2}\right)$
  iii. $P\left(-\frac{1}{2} < X \leq \frac{1}{2}\right)$
  iv. $P(X > 1)$
d. What is the cdf of X.
e. Find the probabilities in (b) using the cdf.

Q6) Suppose continuous r.v. X has density function $f(x) = \begin{cases} cx^2 & ; for 1 < x < 2 \\ 0 & ; otherwise \end{cases}$

a. Find the value of the constant c.
b. Find $P\left(X \geq \frac{3}{2}\right)$.
c. Find the cumulative distribution function of X.
d. Find $P\left(X \geq \frac{3}{2}\right)$ using the cdf.

Q7) A system can function for a random amount of time X. If the density of X is given (in units of months) by

$$f(x) = Cxe^{-x/2}\ ; x > 0$$

a. What is the probability that the system functions for at least 5 months.
b. What is the probability that the system functions from 3 to 6 months.
c. What is the probability that the system functions less than 1 month.

Q8) The cumulative distribution function of a continuous r.v. Y is given by

$$F(y) = \begin{cases} 0 & ; for\ \ y \leq 3 \\ 1 - \dfrac{9}{y^2} & ; for\ \ y > 3 \end{cases}$$

Find

a. $P(Y \leq 5)$.
b. $P(Y > 8)$.
c. the pdf of Y.

Q9) If the density function of the continuous r.v. X is $f(x) = \begin{cases} x & ;\ 0 < x < 1 \\ 2 - x & ;\ 1 \leq x < c \\ 0 & ;\ o.w. \end{cases}$. Find

a. The value of c.
b. The cumulative distribution function of X.
c. $P(0.8 < X < 0.6c)$.

EXPECTATIONS INVOLVING INDEPENDENT R.V.'S AND MOMENT GENERATING FUNCTIONS

Q1) Let X_1, X_2 and X_3 be independent r.v.'s with means 4, 9, 3 and variances 3, 7, 5 respectively. For $Y=2X_1-3X_2+4X_3$ and $Z=X_1+2X_2-X_3$, find:

a. E(Y) and E(Z).
b. V(Y) and V(Z).

Q2) If X and Y are independent r.v.'s with E(X)=3, E(Y)=5, V(X)=2, and V(Y)=5, find:

a. E(XY)
b. $E(X^2Y)$

Q3) Let X and Y are independent r.v's with p.d.f $f(x) = e^{-x}; x > 0,\ f(y) = e^{-y};\ y > 0$, find :

a. E(X) and V(X).
b. E(Y) and V(Y).
c. E(XY).
d. $E(X^2 Y^3)$.

Q4) Find the moment generating function of X If you know that $f(x) = 2e - 2x, x > 0$.

Q5) Suppose independent r.v.'s X and Y are such that $M_{X+Y}(t) = \frac{e^{2t}-1}{2t-t^2}$. If $f(x) = \lambda e^{-\lambda x}\ ; x > 0$, what is the distribution of Y.

Q6) X and Y are independent and identically distributed with $M(t) = e^{3t+t^2}$. Find the mgf of Z=2X-3Y+4.

Q7) Suppose X has $M_X(t) = e^{3t+t^2}$. Find the mgf of $Z = \frac{1}{4}(X-3)$ and use it to find the mean and variance of Z.

Q8) Suppose X is a r.v. for which the mgf is $M_X(t) = \frac{1}{4}(3e^t + e^{-t}), -\infty < t < \infty$.

a. Find the mean and variance of X.

Q9) Let $f(x) = 1\ ; 0 \le x \le 1$. Use the moment generating function technique to find the moment generating function of Y=aX+b where a and b are constant.

Q10) Let $f(x) = e^{-x}\ ; x > 0$, find the mgf of Z=3-2X.