Q2.25. Refer to Airfreight breakage Problem 1.21.
a. Set up the ANOVA table. Which elements are additive?
b. Conduct an *F* test to decide whether or not there is a linear association between the number of times a carton is transferred and the number of broken ampules; control the α risk at 0.05. State the alternatives, decision rule, and conclusion.
c. Obtain the *t statistic for the test in part (b) and demonstrate numerically its equivalence to the *F** statistic obtained in part (b).**

$\bar{X} = 1, \bar{Y} = 14.2, S_{XY} = 40, \quad S_{XX} = 10$

$S_{YY} = 177.6, \qquad b_0 = 10.2, \qquad b_1 = 4$

$SSTo = S_{YY}$

$SSR = b_1^2 S_{XX}$

ANOVA TABLE

Source of Variation	d.f	SS	MS	F	p-value
Regression	1	SSR== $4^2 \times 10$ $= 160$	$MSR = 160$	$\frac{160}{2.2} = 72.72$	0.00
Error	8	SSE= 17.6	$MSE = \frac{17.6}{8} = 2.2$		
Total	9	SSTo= 177.6			

$\alpha = 0.05$

1. Hypothesis

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

2. Test statistic

$F^* = 72.72$

3. Decision: Reject H_0if $F^* > F_{(1-\alpha,1,n-2)},\ 72.72 > F_{(0.95,1,8)} = 5.31$

Then reject H_0

p-value=$P\left(F_{(1,n-2)} > F^*\right) = \left(1 - P\left(F_{(1,8)} < 72.72\right)\right) = (1 - 0.9999) = 0.0001 < 0.05$

, then we reject H_0.

Analysis of Variance

Source	DF	Adj SS	AdjMS	F-Value	P-Value
Regression	1	160.000	160.000	72.73	0.000
Xi	1	160.000	160.000	72.73	0.000
Error	8	17.600	2.200		
Lack-of-Fit	2	0.933	0.467	0.17	0.849
Pure Error	6	16.667	2.778		
Total	9	177.600			

$t^* = 8.528, (t^*)^2 = (8.528)^2 = 72.72 = F^*$

Chapter 2

2.13 Refer to Grade point average.

Calculate R^2. What proportion of the variation in Y is accounted for by introducing X into the regression model?

From page 98

$$\bar{X} = 24.725, \sum_{i=1}^{n=120}(X_i - \bar{X})^2 = 2379.925$$

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	1	SSR=3.588	3.5878	9.24	0.003
Xi	1	3.588	3.5878	9.24	0.003
Error	118	SSE=45.818	MSE=0.3883		
Lack-of-Fit	19	6.486	0.3414	0.86	0.632
Pure Error	99	39.332	0.3973		

Total 119 SSTo=49.405

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.623125	**7.26%**	**6.48%**	**3.63%**

$$R^2 = \frac{SSR}{SSTo} = \frac{3.588}{49.405} = 0.0726$$

$$R^2 = 1 - \frac{SSE}{SSTo} = 1 - \frac{45.818}{49.405} = 1 - 0.9274 = 0.0726$$

This means that 7.26% of change in the mean freshman OPA for students is by ACT test score

a. Obtain a 95 percent interval estimate of the mean freshman OPA for students whose ACT test score is 28. Interpret your confidence interval.

From page 76- to 79

$$\widehat{Y_h} = b_0 + b_1 X_h$$

$$s^2(\widehat{Y_h}) = MSE\left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right)$$

$$\widehat{Y_h} \pm t\left(1 - \frac{\alpha}{2}; n-2\right) s(\widehat{Y_h})$$

$$\alpha = 0.05, \frac{\alpha}{2} = 0.025$$

At $X_h = 28$

$$\widehat{Y_h} = 2.114 + 0.0388\,(28) = 3.2012$$

$$s^2(\widehat{Y_h}) = MSE\left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right)$$

$$s^2(\widehat{Y_h}) = MSE\left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right)$$

$$= \mathbf{0.3883}\left(\frac{1}{120} + \frac{(28 - 24.725)^2}{2379.925}\right) = 0.004986$$

$$s(\widehat{Y_h}) = \sqrt{0.007776} = 0.0706$$

$$t\left(1 - \frac{\alpha}{2}; n - 2\right) = t(0.975; 118) = 1.9807$$

$$3.22012 \pm 1.9807(0.0706)$$

$$3.0614 < E(Y_h) < 3.3410$$

b. Mary Jones obtained a score of 28 on the entrance test. Predict her freshman OPA-using a 95 percent prediction interval. Interpret your prediction interval.

$$s^2(\widehat{Y_{new}}) = MSE\left(1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right)$$

$$\widehat{Y_h} \pm t\left(1 - \frac{\alpha}{2}; n - 2\right) s(\widehat{Y_{new}})$$

$$s^2(\widehat{Y_{new}}) = MSE\left(1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right)$$

$$= \mathbf{0.3883}\left(1 + \frac{1}{120} + \frac{(28 - 24.725)^2}{2379.925}\right) = 0.39328$$

$$s(\widehat{Y_{new}}) = 0.6271$$

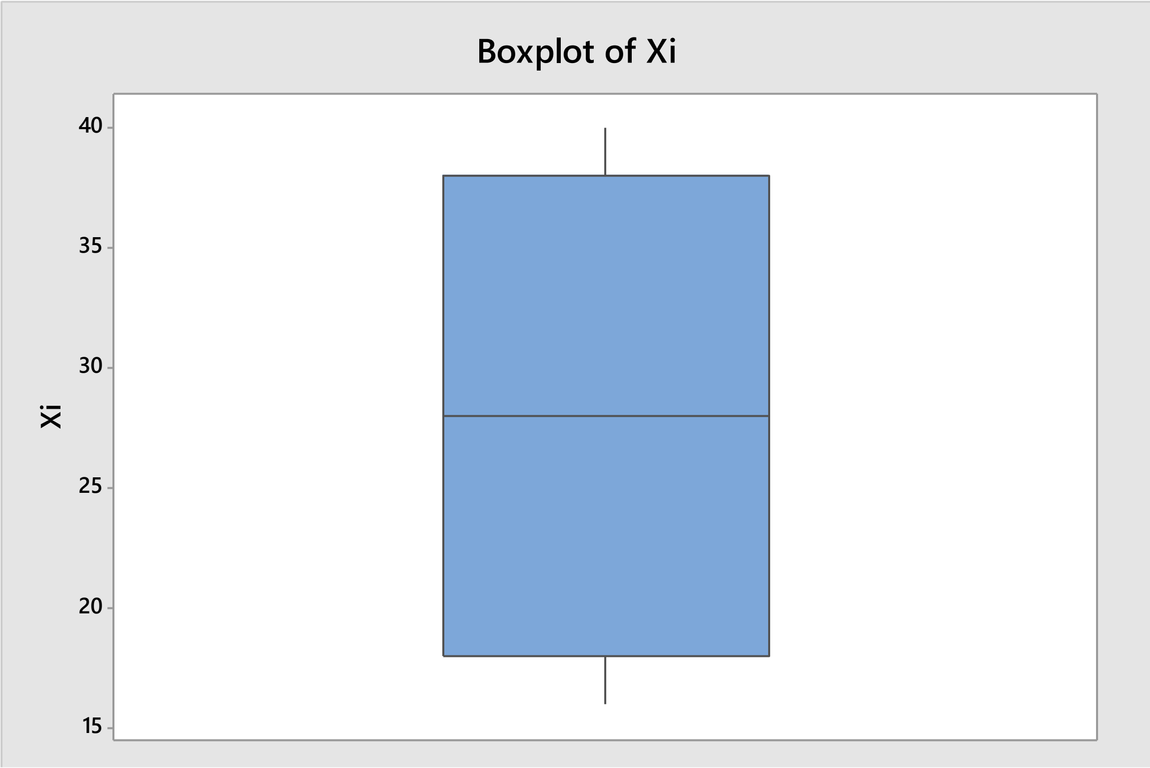
$$3.22012 \pm 1.9807(0.6271)$$

$$1.9594 < Y_{h(new)} < 4.4430$$

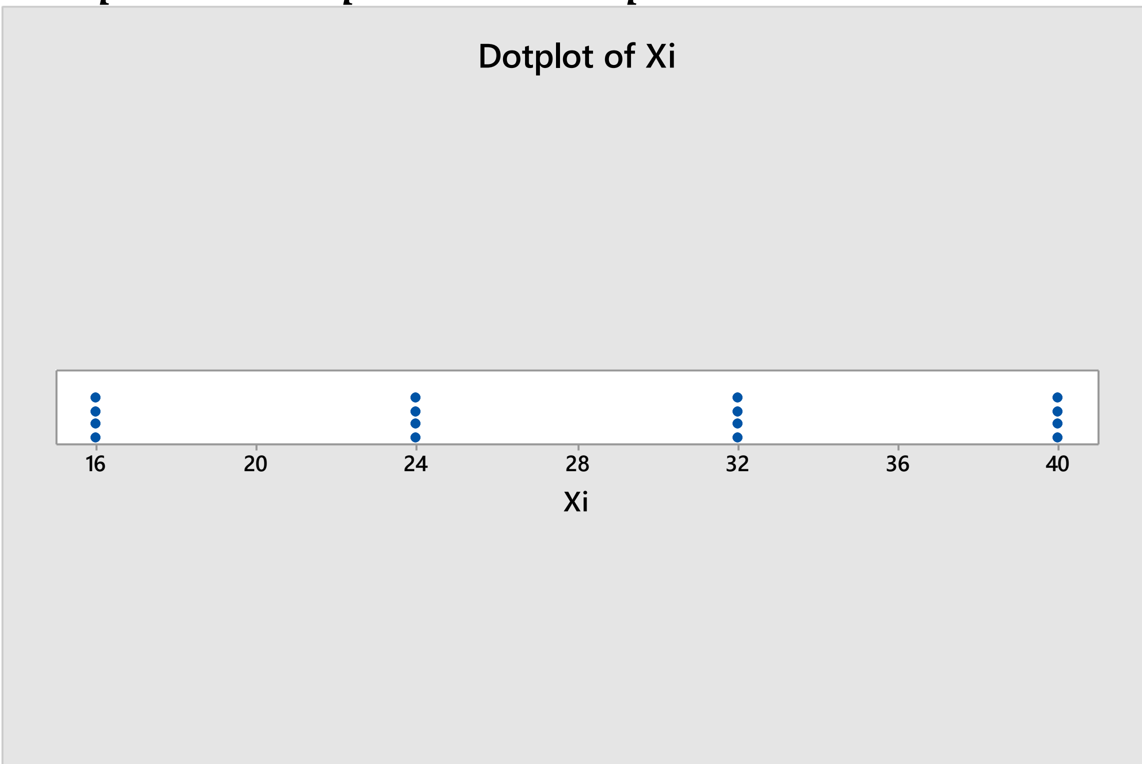
```
Predicted Values for New Observations

New
Obs     Fit  SE Fit       95% CI             95% PI
  1  3.2012  0.0706  (3.0614, 3.3410)(1.9594, 4.4431)

Values of Predictors for New Observations

New
Obs    Xi
  1  28.0
```

Refer to Plastic hardness.

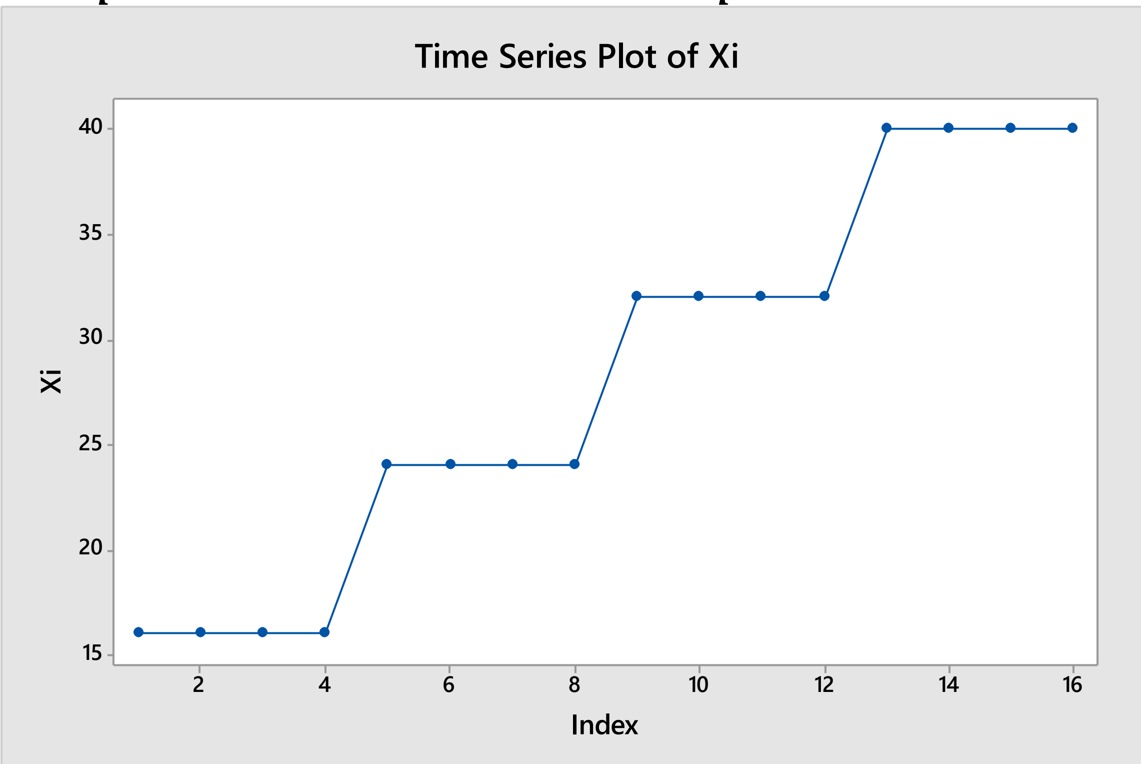
$Graph \rightarrow Boxplot \rightarrow simple \rightarrow X \rightarrow ok$



$Graph \rightarrow Dotplot \rightarrow simple \rightarrow X \rightarrow ok$

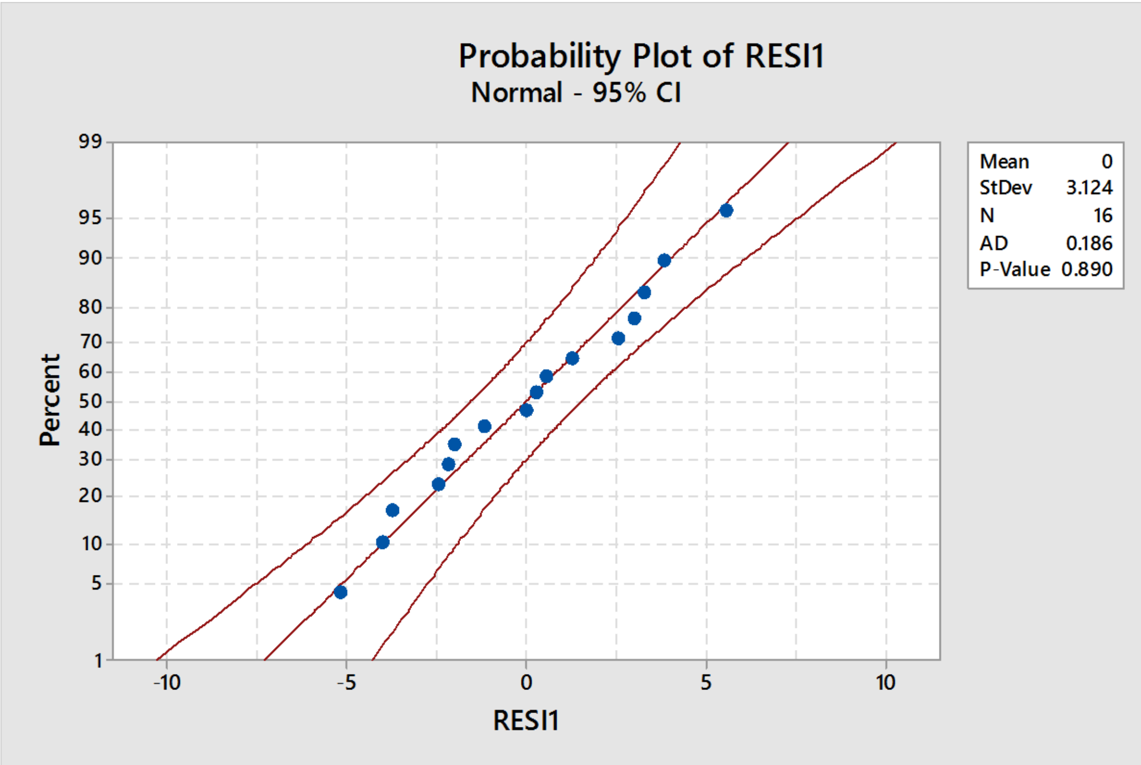


$$Graph \rightarrow time\ series \rightarrow simple \rightarrow X \rightarrow ok$$



For test normality of residuals

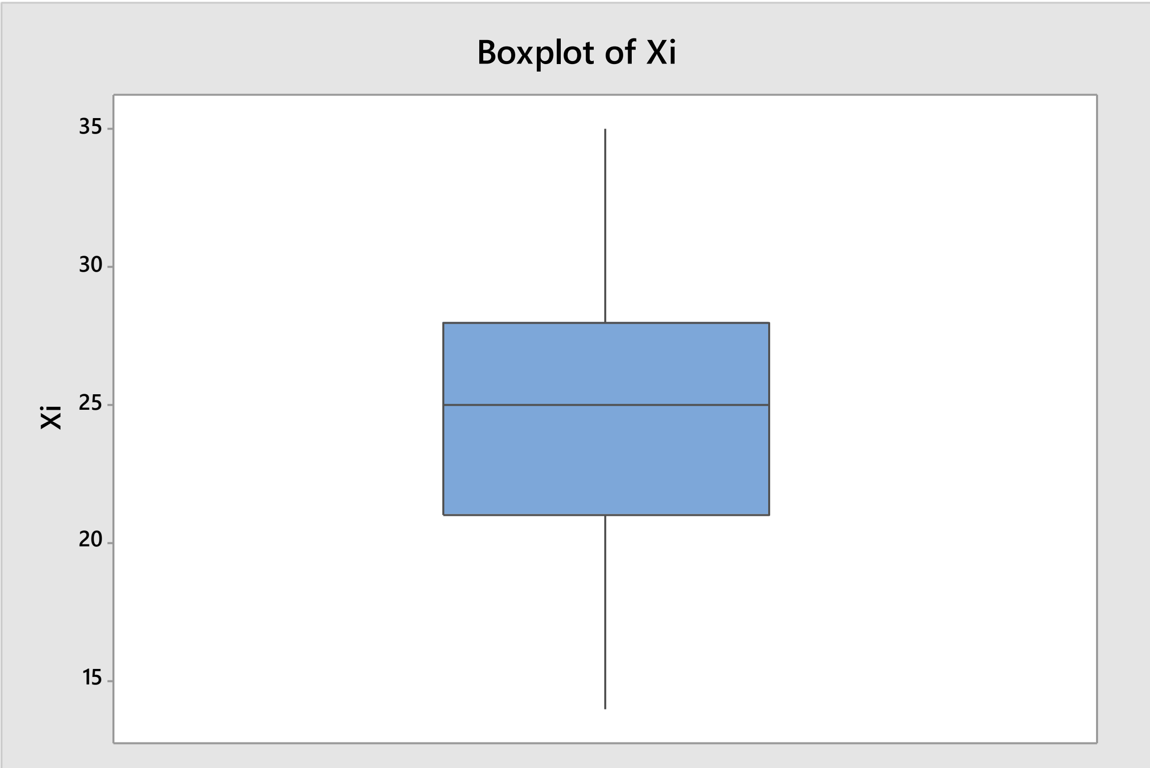
$$Graph \rightarrow probability\ plot \rightarrow single \rightarrow (distribution\ Normal) \rightarrow X \rightarrow ok$$



If p-value >0.05 , then it is normal

Refer to Grade point average.

$Graph \rightarrow Boxplot \rightarrow simple \rightarrow X \rightarrow ok$



$$Graph \rightarrow Dotplot \rightarrow simple \rightarrow X \rightarrow ok$$