Tutorial 3

Digital Signals and Systems

Exercise 1

Calculate the first eight sample values and sketch each of the following sequences:

a. $x(n) = -2\delta(n-5)$

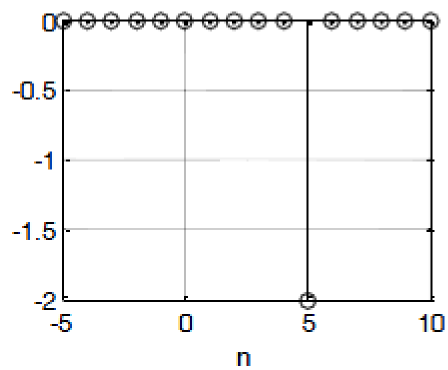
b. $x(n) = 5u(n-2)$

c. $x(n) = 0.5^n u(n)$

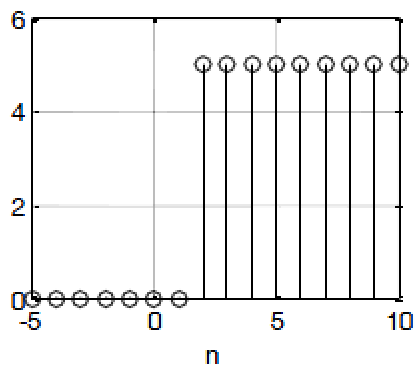
d. $x(n) = 5\sin(0.2\pi n)u(n)$

Solution 1

a.



b.



c.

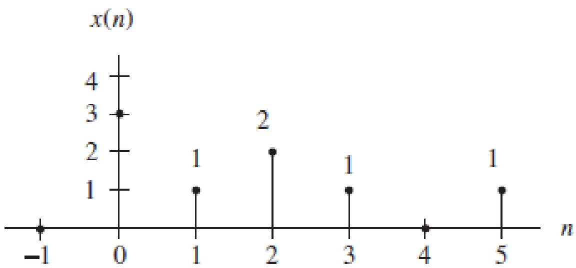
n	0	1	2	3	4	5	6	7
$x(n)$	1.000	0.5000	0.2500	0.1250	0.0625	0.0313	0.0156	0.0078

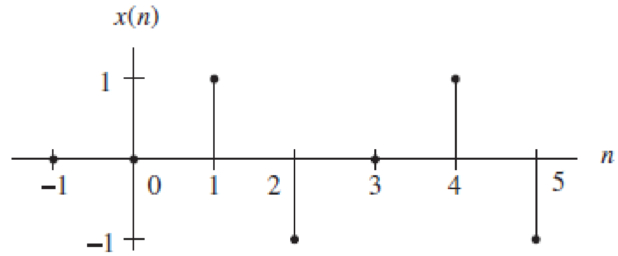
d.

n	0	1	2	3	4	5	6	7
$x(n)$	0.0000	2.9389	4.7553	4.7553	2.9389	0.0000	-2.9389	-4.7553

Exercise 2

Given the digital signals $x(n)$ in Figures write an expression for each digital signal using the unit-impulse sequence and its shifted sequences.





Solution 2

a. $x(n) = 3\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3) + \delta(n-5)$

b. $x(n) = \delta(n-1) - \delta(n-2) + \delta(n-4) - \delta(n-5)$

Exercise 3

Assume that a DS processor with a sampling time interval of 0.01 second converts the following analog signals $x(t)$ to a digital signal $x(n)$; determine the digital sequence for each of the analog signals.

a. $x(t) = e^{-50t}u(t)$

b. $x(t) = 5\sin(20\pi t)u(t)$

Solution 3

a. $x(n) = e^{-0.5n}u(n) = (0.6065)^n u(n)$ b. $x(n) = 5\sin(0.2\pi n)u(n)$

Exercise 4

Determine whether the following systems are linear

a. $y(n) = 5x(n) + 2x^2(n)$

b. $y(n) = x(n-1) + 4x(n)$

c. $y(n) = 4x^3(n-1) - 2x(n)$

Solution 4

a. Let $y_1(n) = 5x_1(n) + 2x_1^2(n)$, $y_2(n) = 5x_2(n) + 2x_2^2(n)$

$$y_1(n) + y_2(n) = 5x_1(n) + 2x_1^2(n) + 5x_2(n) + 2x_2^2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 5x(n) + 2x^2(n) = 5\left(x_1(n) + x_2(n)\right) + 2\left(x_1(n) + x_2(n)\right)^2$$

$$= 5x_1(n) + 5x_2(n) + 2x_1^2(n) + 2x_2^2(n) + 4x_1(n)x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

b. Let $y_1(n) = x_1(n-1) + 4x_1(n)$, $y_2(n) = x_2(n-1) + 4x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = y(n-1) + 4x(n) = \left(x_1(n-1) + x_2(n-1)\right) + 4\left(x_1(n) + x_2(n)\right)$$

$$= x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

c. Let $y_1(n) = 4x_1^3(n) - 2x_1(n)$, $y_2(n) = 4x_2^3(n) - 2x_2(n)$

$$y_1(n) + y_2(n) = 4x_1^3(n) - 2x_1(n) + 4x_2^3(n) - 2x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$y(n) = 5x(n) + 2x^2(n) = 4\left(x_1(n) + x_2(n)\right)^3 - 2\left(x_1(n) + x_2(n)\right)$$

$$= 4x_1^3(n) + 8x_1^2(n)x_2(n) + 8x_1(n)x_2^2(n) + 4x_2^3(n) - 2x_1(n) - 2x_2(n)$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

Exercise 5

Determine whether the following linear systems are time-invariant.

a. $y(n) = -5x(n-10)$

b. $y(n) = 4x(n^2)$

Solution 5

a. For $x_1(n) = x(n-n_0)$, $y_1(n) = -5x_1(n-10) = -5x(n-10-n_0)$

Since $y(n-n_0) = -5x\big((n-n_0)-10\big) = -5x(n-10-n_0) = y_1(n)$

The system is time invariant.

b. For $x_2(n) = x(n-n_0)$ so that $x_2(n^2) = x(n^2-n_0)$, $y_2(n) = 4x_2(n^2) = 4x_2(n^2-n_0)$

Since shifting $y(n-n_0) = 4x\big((n-n_0)^2\big) = 4x(n^2-2nn_0+n_0^2) \neq y_2(n)$

The system is time variant.

Exercise 6

Determine which of the following linear systems is causal.

a. $y(n) = 0.5x(n) + 100x(n-2) - 20x(n-10)$

b. $y(n) = x(n+4) + 0.5x(n) - 2x(n-2)$

Solution 6

a. Since the output is depending on the current input and past inputs, the system is causal.

b. Since the output is depending on the future input $x(n+4)$, the system is a non-causal system.

Exercise 7

Find the unit-impulse response for each of the following linear systems.

a. $y(n) = 0.5x(n) - 0.5x(n-2)$; for $n \geq 0$, $x(-2) = 0$, $x(-1) = 0$

b. $y(n) = 0.75y(n-1) + x(n)$; for $n \geq 0$, $y(-1) = 0$

c. $y(n) = -0.8y(n-1) + x(n-1)$; for $n \geq 0$, $x(-1) = 0$, $y(-1) = 0$

Solution 7

a. $h(n) = 0.5\delta(n) - 0.5\delta(n-2)$ b. $h(n) = (0.75)^n$; $n \geq 0$

c. $h(n) = 1.25\delta(n) - 1.25(-0.8)^n$; $n \geq 0$

Exercise 8

Determine the stability for each of the following linear systems.

a. $y(n) = \sum_{k=0}^{\infty} 0.75^k x(n-k)$

b. $y(n) = \sum_{k=0}^{\infty} 2^k x(n-k)$

Solution 8

a. $h(n) = (0.75)^n u(n)$, $S = \sum_{k=0}^{\infty} (0.75)^k = 1/(1-0.75) = 4 =$ finite, the system is stable.

b. $h(n) = (2)^n u(n)$, $S = \sum_{k=0}^{\infty} (2)^k = 1 + 2 + 2^2 + \cdots = \infty =$ infinite, the system is unstable.

Exercise 9

Using the sequence definitions, evaluate the digital convolution

$$h(k) = \begin{cases} 2, & k = 0, 1, 2 \\ 1, & k = 3, 4 \\ 0 & elsewhere \end{cases} \quad and\ x(k) = \begin{cases} 2, & k = 0 \\ 1, & k = 1, 2 \\ 0 & elsewhere \end{cases}$$

a. using the graphical method;

b. using the table method;

c. applying the convolution formula directly.

Solution 9

$y(0) = 4$, $y(1) = 6$, $y(2) = 8$, $y(3) = 6$, $y(4) = 5$, $y(5) = 2$, $y(6) = 1$,
$y(n) = 0$ for $n \geq 7$

k	-4	-3	-2	-1	0	1	2	3	4	5	6	
x(k)					**2**	**1**	**1**					
h(-k)	1	1	2	2	2							y(0)=4
h(1-k)		1	1	2	2	2						y(1)=6
h(2-k)			1	1	2	2	2					y(2)=8
h(3-k)				1	1	2	2	2				y(3)=6
h(4-k)					1	1	2	2	2			y(4)=5
h(5-k)						1	1	2	2	2		y(5)=2
h(6-k)							1	1	2	2	2	y(6)=1