MATH203 Calculus

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Taylor series

Taylor series

If a function has derivatives of all order at $x = c$, then the series $f(c) + (x-c)f'(c) + \frac{(x-c)^2}{2!}f''(c) + \cdots + \frac{(x-c)^n}{n!}f^n(c) + \cdots = \sum_{n=0}^{\infty} \frac{(x-c)^n}{n!} f^n(c)$ is called Taylor series for $f(x)$ at $x = c$.

Maclaurin series

If a function has derivatives of all order at $x = 0$, then the series $f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots + \frac{x^n}{n!}f^n(0) + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} f^n(0)$ is called Maclaurin series for $f(x)$ at $x = 0$.

Examples

Find Maclaurin series which represents the following functions for every real number x:

(1): $f(x) = e^x$

(2): $f(x) = \sin(x)$

(3): $f(x) = \cos(x)$

(4): $f(x) = \ln(1+x)$

(5): $f(x) = \tan^{-1}(x)$

(6): $f(x) = \sinh(x)$

(7): $f(x) = \cosh(x)$

(8): $f(x) = x^2 \sin(x)$

(9): $f(x) = \cos(x^2)$

(10) $f(x) = \sin(x)$ at $x = \frac{\pi}{4}$

Solution: